

A generalized nonlocal gravity framework based on Poincaré gauge theory

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We describe a framework for a generalized nonlocal gravity theory inspired by Poincaré gauge theory. Our theory provides a unified description of previous nonlocal extensions of Einstein’s theory of gravitation, in particular it allows for a clear geometrical foundation. Furthermore, it incorporates recent simplifications for the ansatz of the nonlocality, which should allow for a systematic study of the impact of nonlocal concepts on observations.

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I. INTRODUCTION

Based on our recent re-analysis [1] of a nonlocal gravity (NLcG) theory, originally proposed by Mashhoon [2], we here suggest a nonlocal generalization inspired by the structure of Poincaré gauge gravity (PGT).

In NLcG theory, gravity is assumed to be history dependent, i.e. the gravitational interaction has an additional feature of nonlocality in the sense of an influence (“memory”) from the past that endures. NLcG exhibits some very promising properties – for example providing a possible solution of the dark matter problem [3–7]. The theory is built upon an ansatz for the so-called nonlocality tensor, leading to a set of integro-differential field equations.

Our new framework theory, termed nonlocal Poincaré gauge gravity (NLcPGT), draws from its conceptually clear underpinning in the form of PGT, which has been well established in a gauge gravity context over the last decades. In particular, the new framework allows for a unified description of the already known nonlocal gravity extensions in the literature. The present work can be seen as the refinement and generalization of the suggestion made in the appendix of [4].

The structure of the paper is as follows: In section II we give a condensed account of the basic structures in Poincaré gauge theory. This is followed by our suggestion for the nonlocal generalization of PGT in III. In IV we discuss some special limiting cases of the theory, in particular a nonlocal version of Einstein-Cartan theory. We conclude our paper in section V with a discussion and outlook. An overview of our notation can be found in table I in appendix A.

II. POINCARÉ GAUGE THEORY

The gauge approach in field theory has a long history and detailed reviews of the development of the field can be found in [8–14]. The basic idea of a gauge theory of gravity based on the Poincaré symmetry group $G = T_4 \rtimes SO(1, 3)$ may be sketched as follows: The invariance of the action under an N -parameter group of field transformations yields, via the Noether theorem, N conserved currents. When the parameters are allowed to be functions of spacetime coordinates, one needs to introduce N gauge fields, which are coupled to the Noether currents, to preserve the invariance under the local (gauge) symmetry. In accordance with the general Yang-Mills-Utiyama-Kibble scheme, the 10-parameter Poincaré group gives rise to the 10-plet of the gauge potentials which are identified with the coframe $\vartheta^\alpha = e_i^\alpha dx^i$ (4 one-form potentials corresponding to the translation subgroup T_4) and the local connection $\Gamma^{\alpha\beta} = -\Gamma^{\beta\alpha} = \Gamma_i^{\alpha\beta} dx^i$ (6 one-form potentials for the Lorentz subgroup $SO(1, 3)$).

Compared to general relativity (GR), in which the gravitational field equations are second-order local partial differential equations (PDEs), in the gauge approach to gravity the gravitational field equations take the form of first-order local PDEs [15–19]. One can then extend the first-order local field equations to *nonlocal* ones via the introduction of a “constitutive” kernel as in the phenomenological electrodynamics of media [20]. The corresponding nonlocal generalization of Einstein’s theory of gravitation based on the teleparallel equivalent of GR was recently developed in [3, 21, 22]. The gauge theory of spacetime translations represents a degenerate subcase of the gauge theory of the Poincaré group. Here we propose a consistent generalization of the nonlocal translational gauge theory to the PGT.

The gravitational gauge field Lagrangian density $\mathcal{L}_{\text{grav}} = \mathcal{L}_{\text{grav}}(e_i^\alpha, T_{ij}^\alpha, R_{ij}^{\alpha\beta})$ of the underlying Riemann-Cartan spacetime is a function of the coframe

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e_i^α , the torsion

$$T_{ij}^\alpha := \partial_i e_j^\alpha - \partial_j e_i^\alpha + \Gamma_{i\beta}^\alpha e_j^\beta - \Gamma_{j\beta}^\alpha e_i^\beta, \quad (1)$$

and the curvature

$$R_{ij}^{\alpha\beta} := \partial_i \Gamma_j^{\alpha\beta} - \partial_j \Gamma_i^{\alpha\beta} + \Gamma_{i\gamma}^\beta \Gamma_j^{\alpha\gamma} - \Gamma_{j\gamma}^\beta \Gamma_i^{\alpha\gamma}. \quad (2)$$

The matter Lagrangian \mathcal{L}_{mat} depends on the matter field(s) Ψ which are minimally coupled to gravity. Then the total Lagrangian density reads

$$\mathcal{L}_{\text{tot}} = \mathcal{L}_{\text{grav}}(e_i^\alpha, T_{ij}^\alpha, R_{ij}^{\alpha\beta}) + \mathcal{L}_{\text{mat}}(e_i^\alpha, \Psi, D_i \Psi). \quad (3)$$

Defining the two gravitational field excitations

$$\mathcal{H}^{ij}{}_\alpha := -2 \frac{\partial \mathcal{L}_{\text{grav}}}{\partial T_{ij}^\alpha}, \quad (4)$$

$$\mathcal{H}^{ij}{}_{\alpha\beta} := -2 \frac{\partial \mathcal{L}_{\text{grav}}}{\partial R_{ij}^{\alpha\beta}}, \quad (5)$$

we derive the two field equations from the variation of \mathcal{L}_{tot} with respect to e_i^α and $\Gamma_i^{\alpha\beta}$

$$D_j \mathcal{H}^{ij}{}_\alpha - \mathcal{E}_\alpha{}^i = \mathfrak{T}_\alpha{}^i, \quad (6)$$

$$D_j \mathcal{H}^{ij}{}_{\alpha\beta} - e^j{}_{[\alpha} \mathcal{H}^i{}_{|\beta|]} = \mathfrak{S}_{\alpha\beta}{}^i, \quad (7)$$

where $\mathfrak{T}_\alpha{}^i := \delta \mathcal{L}_{\text{mat}} / \delta e_i^\alpha$ denotes the canonical energy-momentum tensor density of the matter field and $\mathfrak{S}_{\alpha\beta}{}^i := \delta \mathcal{L}_{\text{mat}} / \delta \Gamma_i^{\alpha\beta} = -\mathfrak{S}_{\beta\alpha}{}^i$ denotes the corresponding canonical spin (angular momentum) tensor density (note that these definitions differ slightly from the ones in [23]).

The energy-momentum tensor of the gravitational gauge fields can be expressed as

$$\mathcal{E}_\alpha{}^i = e^i{}_\alpha \mathcal{L}_{\text{grav}} - \mathcal{H}^{jk}{}_\alpha T_{jk}{}^i - \mathcal{H}^{jk}{}_{\alpha\beta} R_{jk}{}^{i\beta}. \quad (8)$$

Equations (3)-(8) represent the general framework for PGT, and particular gravitational models are specified by the explicit form of the gauge Lagrangian. In accordance with the general scheme of a Yang-Mills theory, we assume that the Lagrangian is local and quadratic in the Poincaré gauge field strengths – torsion and curvature. The torsion tensor can be decomposed into the three irreducible pieces which we denote by ${}^{(I)}T_{ij}^\alpha$, $I = 1, 2, 3$, whereas the curvature tensor's six irreducible pieces are denoted by ${}^{(K)}R_{ij}^{\alpha\beta}$, $K = 1, 2, \dots, 6$ (for more details see [18, 23]). Then the quadratic PG Lagrangian reads [14]

$$\begin{aligned} \mathcal{L}_{\text{grav}}^{\text{loc}} &= \frac{\sqrt{-g}}{2\kappa c} \left[(a_0 e^i{}_\alpha e^j{}_\beta - \bar{a}_0 \eta^{ij}{}_{\alpha\beta}/2) R_{ij}^{\alpha\beta} - 2\lambda_0 \right. \\ &\quad \left. - \frac{1}{2} T^{ij}{}_\alpha \sum_{I=1}^3 \left(a_I {}^{(I)}T_{ij}^\alpha - \bar{a}_I {}^{(I)}T^{*ij}{}_\alpha \right) \right] \\ &\quad - \frac{\sqrt{-g}}{4\rho} R^{ij}{}_{\alpha\beta} \sum_{K=1}^6 \left(b_K {}^{(K)}R_{ij}^{\alpha\beta} - \bar{b}_K {}^{(K)}R^{*ij}{}_{\alpha\beta} \right), \end{aligned} \quad (9)$$

where $\kappa = 8\pi G/c^4$ is Einstein's gravitational constant, λ_0 is the cosmological constant, and ρ is the coupling constant of “strong gravity” with dimension $[1/\rho] = [\hbar]$, which is mediated via the propagating Lorentz connection. The constants a_I, \bar{a}_I and b_K, \bar{b}_K are dimensionless and should be of order unity. Note that we put $\bar{a}_2 = \bar{a}_3$, $\bar{b}_2 = \bar{b}_4$ and $\bar{b}_3 = \bar{b}_6$ because some of the quadratic contractions are the same. This most general quadratic Poincaré gravity model encompasses both the parity even and parity odd terms. The corresponding parity odd coupling constants are denoted by the overbars, whereas the dualization of the tensors is denoted by a star: ${}^{(I)}T_{ij}^*\alpha = \frac{1}{2}\eta_{ijkl}{}^{(I)}T^{kl\alpha}$ and ${}^{(K)}R_{ij}^*\alpha\beta = \frac{1}{2}\eta_{ijkl}{}^{(K)}R^{kl\alpha\beta}$.

We compute the excitations from the gravitational field Lagrangian (9) by partial differentiation according to the definitions (4) and (5):

$$\mathcal{H}^{ij}{}_\alpha = \frac{\sqrt{-g}}{\kappa c} \sum_{I=1}^3 \left(a_I {}^{(I)}T^{ij}{}_\alpha - \bar{a}_I {}^{(I)}T^{*ij}{}_\alpha \right), \quad (10)$$

$$\begin{aligned} \mathcal{H}^{ij}{}_{\alpha\beta} &= -\frac{\sqrt{-g}}{\kappa c} (a_0 e^i{}_{[\alpha} e^j{}_{\beta]} - \bar{a}_0 \eta^{ij}{}_{\alpha\beta}/2) \\ &\quad + \frac{\sqrt{-g}}{\rho} \sum_{K=1}^6 \left(b_K {}^{(K)}R^{ij}{}_{\alpha\beta} - \bar{b}_K {}^{(K)}R^{*ij}{}_{\alpha\beta} \right) \\ &= \mathcal{H}^{ij}{}_{\alpha\beta}^{\text{lin}} + \mathcal{H}^{ij}{}_{\alpha\beta}^{\text{qu}}. \end{aligned} \quad (11)$$

This is the quadratic *local* Poincaré gauge theory.

III. NONLOCAL POINCARÉ GAUGE THEORY

In the nonlocal formulation of PGT we will make use of the bitensor formalism [24–26], in particular we adhere to the conventions of [1]. It is worthwhile to mention that we will use a condensed notation (common to the theory of bitensors) in which the point to which the index of a bitensor belongs can be directly read from the index itself; e.g., y_n denotes indices at the spacetime point y . Moreover, in order to distinguish the local frame indices, we use ξ_1, ξ_2, \dots and v_1, v_2, \dots to designate objects with frame indices at the point x or y , in complete analogy to the labels x_1, x_2, \dots and y_1, y_2, \dots used in the holonomic case.

We now generalize the local “constitutive relations” (10) and (11) to nonlocal ones by using an unknown scalar kernel $\mathcal{K}(x, y)$ and the parallel propagator $g_x{}^y$ for

transporting tensors from point x to y :

$$\mathcal{H}^{y_1 y_2}_{v_3} = \frac{1}{\kappa c} \sum_{I=1}^3 \int d^4 x \sqrt{-g(x)} g^{y_1 x_1} g^{y_2 x_2} g_{v_3 \xi_3} \\ \times \mathcal{K}(x, y) \left(a_I {}^{(I)}T_{x_1 x_2}{}^{\xi_3} - \bar{a}_I {}^{(I)}T_{x_1 x_2}{}^{\xi_3} \right), \quad (12)$$

$$\overset{\text{lin}}{\mathcal{H}}^{y_1 y_2}_{v_3 v_4} = -\frac{1}{\kappa c} \int d^4 x \sqrt{-g(x)} g^{y_1 x_1} g^{y_2 x_2} g_{v_3 \xi_3} g_{v_4 \xi_4} \\ \times \mathcal{K}(x, y) \left(a_0 e^{x_1} [\xi_3 e^{x_2} \xi_4] - \bar{a}_0 \eta^{x_1 x_2} \xi_3 \xi_4 / 2 \right), \quad (13)$$

$$\overset{\text{qu}}{\mathcal{H}}^{y_1 y_2}_{v_3 v_4} = \frac{1}{\rho} \sum_{K=1}^6 \int d^4 x \sqrt{-g(x)} g^{y_1 x_1} g^{y_2 x_2} g_{v_3 \xi_3} g_{v_4 \xi_4} \\ \times \mathcal{K}(x, y) \left(b_K {}^{(K)}R_{x_1 x_2}{}^{\xi_3 \xi_4} - \bar{b}_K {}^{(K)}R_{x_1 x_2}{}^{\xi_3 \xi_4} \right), \quad (14)$$

$$\mathcal{H}^{y_1 y_2}_{v_3 v_4} = \overset{\text{lin}}{\mathcal{H}}^{y_1 y_2}_{v_3 v_4} + \overset{\text{qu}}{\mathcal{H}}^{y_1 y_2}_{v_3 v_4}. \quad (15)$$

This nonlocal ansatz (12)–(15) should be used in (8) and in the field equations (6) and (7). In this way, we have a set of 16 + 24 integro-differential equations in terms of the variables e_i^α , $\Gamma_i^{\alpha\beta}$ and Ψ .

IV. SPECIAL CASES

A. Nonlocal Einstein-Cartan theory

The torsion and the curvature square terms are absent when the coupling constants $a_I = 0$, $b_K = 0$, $\bar{a}_I = 0$, and $\bar{b}_K = 0$ vanish; in this case one recovers the Einstein-Cartan-(Holst) model which is characterized by the parity even a_0 and parity odd \bar{a}_0 coupling constants. Conventionally, one puts $a_0 = 1$ and $\bar{a}_0 = 1/\xi$, where ξ is called a Barbero-Immirzi parameter [27, 28].

In the absence of matter sources with spin, the resulting nonlocal Einstein-Cartan-(Holst) theory is described by the constitutive relation (13) and (15). It represents a version of Hehl-Mashhoon nonlocal gravity theory with similar physical properties.

B. Nonlocal teleparallel theory

We can recover the original nonlocal teleparallel gravity theory NLcG [2] when the distant parallelism condition $R_{ij}^{\alpha\beta} = 0$ is assumed. Then only the second line

in the gravitational Lagrangian (9) is nontrivial, and we arrive at a generalized nonlocal teleparallel gravity with an account of parity odd terms.

Strictly speaking, the corresponding nonlocal constitutive law (12) implements the improvements from [1], i.e. it avoids the unjustified complexity of the original ansatz for the nonlocality in [2], and at the same time it maintains full compatibility at the lowest orders. In particular, our choice of the nonlocality is much more natural from the viewpoint of relativistic multipolar schemes [29, 30], since it avoids the emergence of derivatives of the world function as in [21], which do not have a straightforward interpretation. Furthermore, by making use of the parallel propagator in the new ansatz (12), one can expect that our new constitutive law would eventually lead to the possibility of deriving exact solutions in the framework of NLcG.

V. DISCUSSION AND CONCLUSIONS

We presented a new nonlocal version of Poincaré gauge theory. The theory can be thought of as the generalization of the recently simplified version of NLcG discussed in [1]. It incorporates all previously known versions of Mashhoon's original theory from [2], and can be viewed as a unified, and at the same time simplified version of the original theory (in the sense how the nonlocality is implemented within the theory). In particular, it specializes to the nonlocal version of subclasses of PGT like, for example, Einstein-Cartan theory.

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Appendix A: Notations and conventions

Table I contains a brief overview over the symbols used throughout the work.

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TABLE I. Directory of symbols.

Symbol	Explanation
Geometrical quantities	
g_{ab}	Metric
$\Gamma_i^{\alpha\beta}$	Lorentz connection
e_i^α	Coframe
T_{ij}^α	Torsion
$R_{ij}^{\alpha\beta}$	Curvature
η_{ijkl}	Totally antisymm. Levi-Civita tensor
$g_{x_0}^{y_0}$	Parallel propagator
Misc	
$\mathcal{L}_{\text{grav}}, \mathcal{L}_{\text{mat}}, \mathcal{L}_{\text{tot}}$	(Gravitational, matter, total) Lagrangian
Ψ	Matter field(s)
$\mathcal{H}^{ij}_\alpha, \mathcal{H}^{ij}_{\alpha\beta}$	Gravitational field excitations
\mathfrak{T}_α^i	Canonical matter energy-momentum
$\mathfrak{S}_{\alpha\beta}^i$	Canonical spin (angular momentum)
\mathcal{E}_α^i	Gauge field energy-momentum
$\mathcal{K}(x, y)$	Causal scalar kernel
κ	Einstein's gravitational constant
λ_0	Cosmological constant
ρ	"Strong gravity" coupling constant
$a_I, \bar{a}_I, b_K, \bar{b}_K$	Coupling constants
c	Vacuum speed of light
Operators	
∂_i	Partial derivative
D_i	Covariant derivative

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