Status and prospects of non-Riemannian cosmology

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Why cosmology?

- Local tests in rather good compliance with GR (maybe not on very small scales)
- New effects likely to show up on **large scales** and/or **high energies**
- Observations force us to introduce concepts like **dark matter** and **dark energy** within the standard cosmological model
- So far **no** direct detection of a dark matter particle
- So far **no** convincing theoretical explanation for the dark energy component

Alternative approaches to cosmology

- Scalar fields
  Ratra & Peebles 1988, Wetterich 1988
- Time varying cosmological constant
  Oezer & Taha 1987, Vishwakarma 2001
- K-essence
  Chiba et al. 2001, Armendariz-Picon et al. 2000
- Phantom energy
  Caldwell 2002
- Chaplygin gas
  Barrow 1990, Hassaine et al. 2001, Fabris et al. 2002
- Cardassian expansion
  Freese & Lewis 2002, Zhu & Fujimoto 2002
- Brane world models
  Randall & Sundrum 1999, Deffayet et al. 1999
- Non-symmetric gravity
  Moffat 1997, 2001, 2004
- **Non-Riemannian models**

Why go beyond Riemannian gravity?

- Curved spacetime (GR) ~ dynamics of masspoints and light
- Intrinsic properties of particles suggest couplings to new fields
- Simple analogy:

1905 flat spacetime (SR)

1916 curved spacetime (GR)

why not more complex?

Why non-Riemannian gravity ?

- Elementary particles may be classified by the Poincaré group by their **mass** and **spin**
- **Mass** connected with the **translational** part of the Poincaré group
- **Spin** connected with the **rotational** part of the Poincaré group
- Mass and spin are **elementary notions**, each with an analogous standing not reducible to that of the other
- Distributing **mass-energy** and **spin** over space-time leads to the field theoretical notions of an **energy-momentum** tensor and **spin angular momentum** tensor of matter
- In macrophysical limit, mass or **energy-momentum** adds up because of its **monopole** character
- **Intrinsic spin** has **dipole** character and usually averages out in the macroscopic limit (this is one of the reasons why GR is successfull on macroscopic scales)
- In analogy to the coupling of the energy-momentum to the metric one expects that that also spin angular momentum couples to a new quality which is linked to the geometry of spacetime

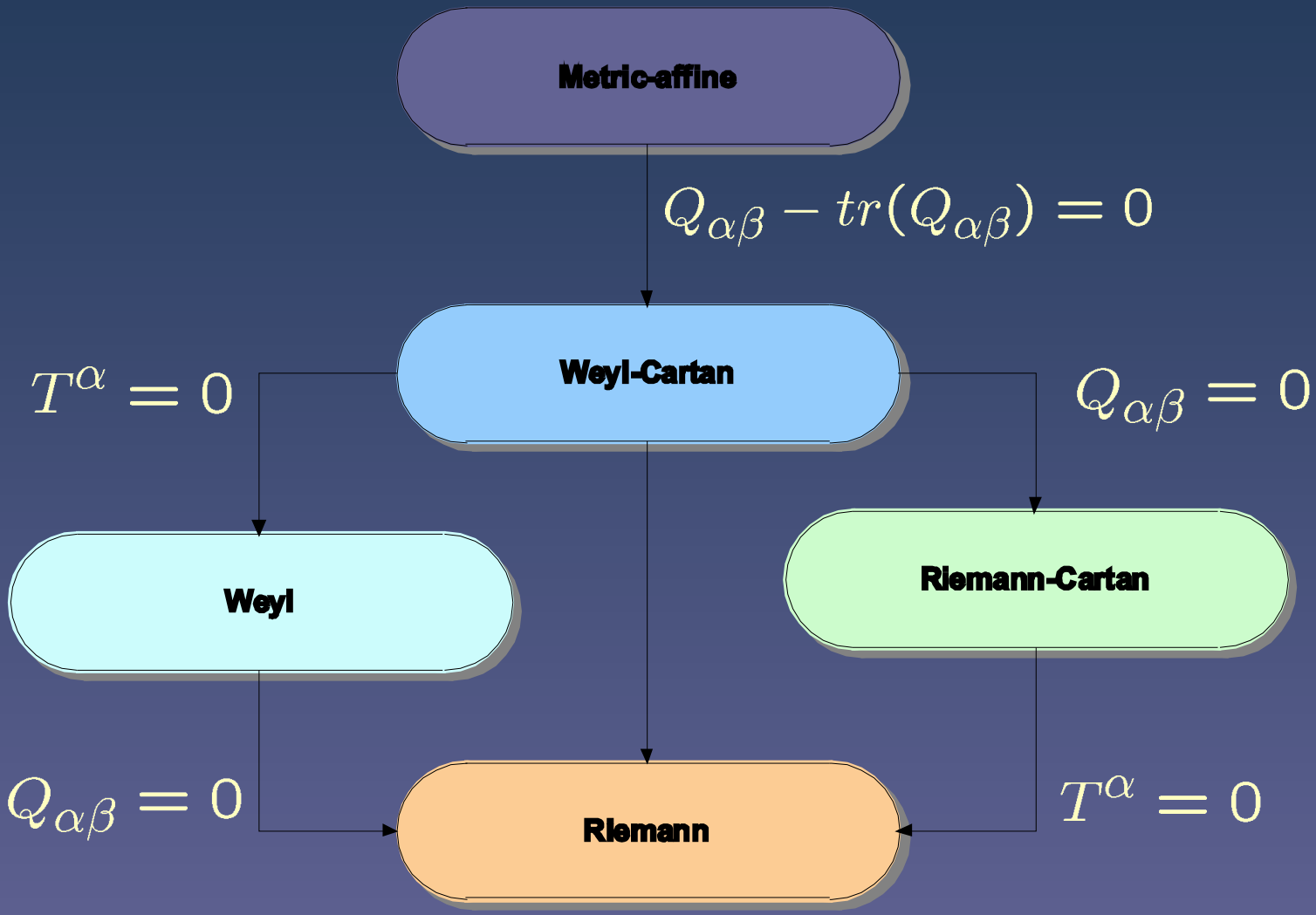
Riemann-Cartan spacetime

Metric-affine gravity (MAG)

Potentials	Field strenghts	Gauge currents	Excitations	Matter currents
$g_{\alpha\beta}$	$Q_{\alpha\beta}$	$m^{\alpha\beta}$	$M^{\alpha\beta}$	$\sigma^{\alpha\beta}$
ϑ^{α}	T^{α}	E_{α}	H_{α}	Σ_{α}
$\Gamma_{\alpha}{}^{\beta}$	$R_{\alpha}{}^{\beta}$	$E^{\alpha}{}_{\beta}$	$H^{\alpha}{}_{\beta}$	$\Delta^{\alpha}{}_{\beta}$

$$L = V_{\mathsf{MAG}}(g_{\alpha\beta}, \vartheta^{\alpha}, Q_{\alpha\beta}, T^{\alpha}, R_{\alpha}{}^{\beta}) + L_{\mathsf{mat}}(g_{\alpha\beta}, \vartheta^{\alpha}, \psi, D\psi)$$

Spacetime types



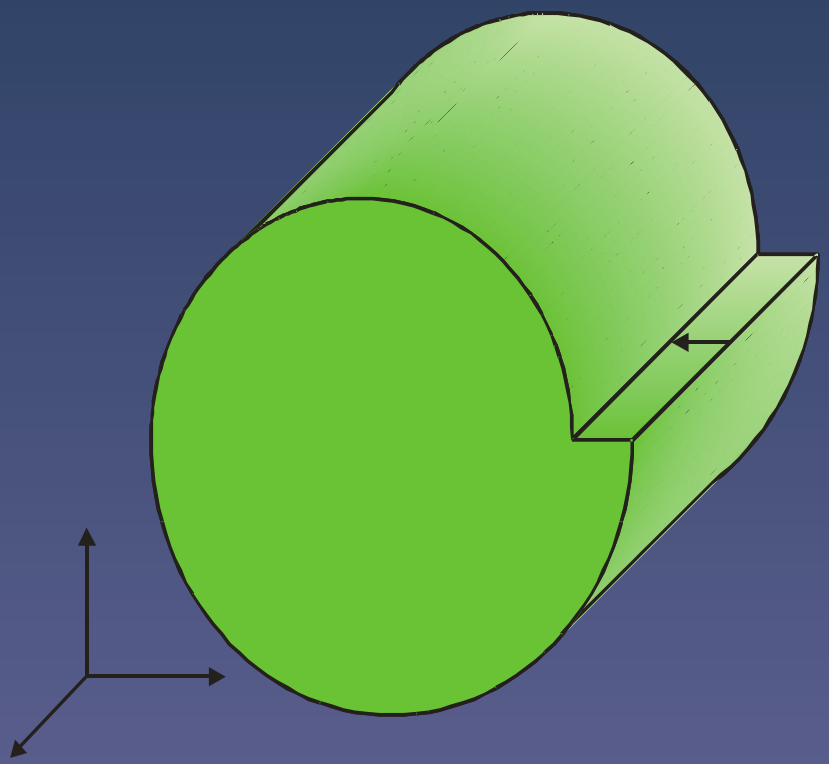
The general MAG connection

$$\Gamma_{\alpha\beta} = \widetilde{\Gamma}_{\alpha\beta} + N_{\alpha\beta} =$$

$$\underbrace{\frac{1}{2}dg_{\alpha\beta} + \left(e_{[\alpha}\rfloor dg_{\alpha]\gamma}\right)\vartheta^{\gamma} + e_{[\alpha}\rfloor C_{\beta]} - \frac{1}{2}\left(e_{\alpha}\rfloor e_{\beta}\rfloor C_{\gamma}\right)\vartheta^{\gamma}}_{\text{Levi-Civita connection}}$$

$$\underbrace{-e_{[\alpha}\rfloor T_{\beta]} + \frac{1}{2}\left(e_{\alpha}\rfloor e_{\beta}\rfloor T_{\gamma}\right)\vartheta^{\gamma} + \frac{1}{2}Q_{\alpha\beta} + \left(e_{[\alpha}\rfloor Q_{\beta]\gamma}\right)\vartheta^{\gamma}}_{\text{Distortion}}$$

Analogy with elasticy theory and theory of defects



Torsion & Nonmetricity ~ Dislocations

$$b^i \sim \int\int dx^k \wedge dx^l \, T^i$$

In some cases torsion can be interpreted as the surface density of the Burgers vector, i.e. it is proportional to the dislocation density of an elastic medium

Kröner 1958, Hehl 1967

Field equations

$$\frac{\delta L_{\mathrm{mat}}}{\delta \psi} = 0$$

0 $$DM^{\alpha\beta} - m^{\alpha\beta} = \sigma^{\alpha\beta}$$

I $$DH_{\alpha} - E_{\alpha} = \Sigma_{\alpha}$$

II $$DH^{\alpha}{}_{\beta} - E^{\alpha}{}_{\beta} = \Delta^{\alpha}{}_{\beta}$$

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The general MAG Lagrangian

$$
\begin{aligned}
V_{\mathrm{MAG}} \;=\; & \frac{1}{2\kappa}[-a_0 R^{\alpha\beta}\wedge\eta_{\alpha\beta} - 2\lambda\eta + T^{\alpha}\wedge{}^{\star}\left(\sum_{I=1}^{3} a_I\, {}^{(I)}T_{\alpha}\right) \\
& + Q_{\alpha\beta}\wedge{}^{\star}\left(\sum_{I=1}^{4} b_I\, {}^{(I)}Q^{\alpha\beta}\right) \\
& + b_5\left({}^{(3)}Q_{\alpha\gamma}\wedge\vartheta^{\alpha}\right)\wedge{}^{\star}\left({}^{(4)}Q^{\beta\gamma}\wedge\vartheta_{\beta}\right) \\
& + 2\left(\sum_{I=2}^{4} c_I\, {}^{(I)}Q_{\alpha\beta}\right)\wedge\vartheta^{\alpha}\wedge{}^{\star}T^{\beta}] \\
-\frac{1}{2\rho}R^{\alpha\beta} \;\wedge\; & {}^{\star}[\sum_{I=1}^{6} w_I\, {}^{(I)}W_{\alpha\beta} + \sum_{I=1}^{5} z_I\, {}^{(I)}Z_{\alpha\beta} + w_7\vartheta_{\alpha}\wedge\left(e_{\gamma}\rfloor\, {}^{(5)}W^{\gamma}{}_{\beta}\right) \\
& + z_6\vartheta_{\gamma}\wedge\left(e_{\alpha}\rfloor\, {}^{(2)}Z^{\gamma}{}_{\beta}\right) + \sum_{I=7}^{9} z_I\vartheta_{\alpha}\wedge\left(e_{\gamma}\rfloor\, {}^{(I-4)}Z^{\gamma}{}_{\beta}\right)]
\end{aligned}
$$

Hehl et al. 1999

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-\frac{1}{2\rho}R^{\alpha\beta} \;\wedge\; & {}^{\star}[\sum_{I=1}^{6} w_I\, {}^{(I)}W_{\alpha\beta} + \sum_{I=1}^{5} z_I\, {}^{(I)}Z_{\alpha\beta} + w_7\vartheta_{\alpha}\wedge\left(e_{\gamma}\rfloor\, {}^{(5)}W^{\gamma}{}_{\beta}\right) \\
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\end{aligned}
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Hehl et al. 1999

The general MAG Lagrangian

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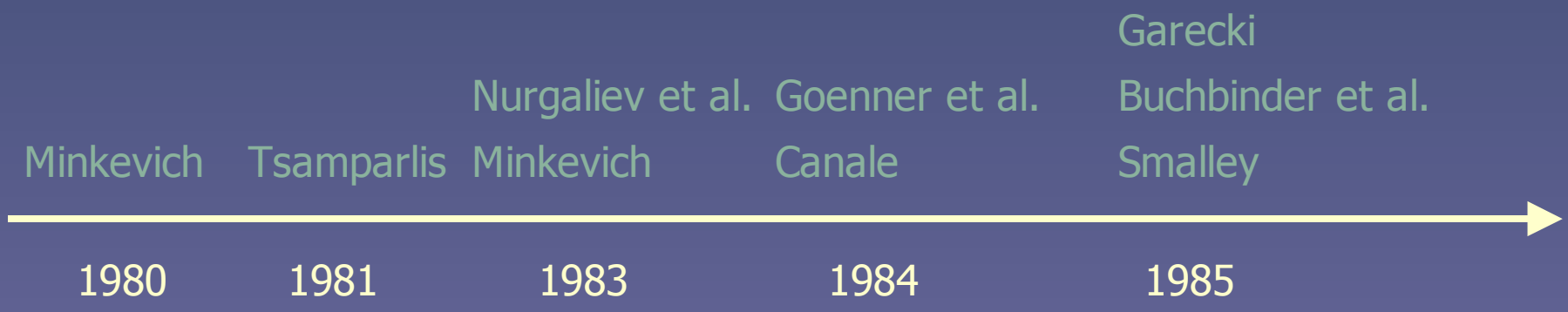
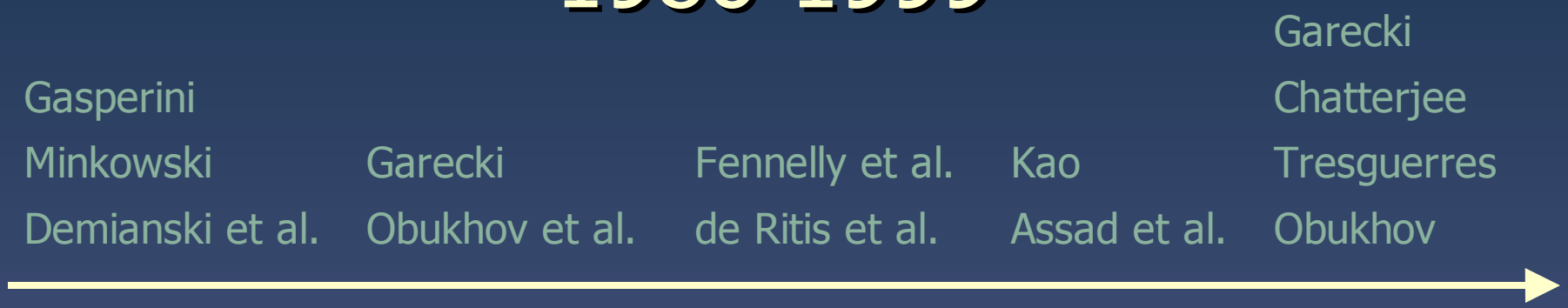
\begin{aligned}
V_{\mathrm{MAG}} \;=\; & \frac{1}{2\kappa}\Big[-a_0 R^{\alpha\beta}\wedge\eta_{\alpha\beta}-2\lambda\eta+T^{\alpha}\wedge{}^{\star}\left(\sum_{I=1}^{3}a_I\,{}^{(I)}T_{\alpha}\right)\\
& +Q_{\alpha\beta}\wedge{}^{\star}\left(\sum_{I=1}^{4}b_I\,{}^{(I)}Q^{\alpha\beta}\right)\\
& +b_5\left({}^{(3)}Q_{\alpha\gamma}\wedge\vartheta^{\alpha}\right)\wedge{}^{\star}\left({}^{(4)}Q^{\beta\gamma}\wedge\vartheta_{\beta}\right)\\
& +2\left(\sum_{I=2}^{4}c_I\,{}^{(I)}Q_{\alpha\beta}\right)\wedge\vartheta^{\alpha}\wedge{}^{\star}T^{\beta}\Big]\\
-\frac{1}{2\rho}R^{\alpha\beta}\;\wedge\;{}^{\star}\Big[& \sum_{I=1}^{6}w_I\,{}^{(I)}W_{\alpha\beta}+\sum_{I=1}^{5}z_I\,{}^{(I)}Z_{\alpha\beta}+w_7\vartheta_{\alpha}\wedge\left(e_{\gamma}\rfloor\,{}^{(5)}W^{\gamma}{}_{\beta}\right)\\
& +z_6\vartheta_{\gamma}\wedge\left(e_{\alpha}\rfloor\,{}^{(2)}Z^{\gamma}{}_{\beta}\right)+\sum_{I=7}^{9}z_I\vartheta_{\alpha}\wedge\left(e_{\gamma}\rfloor\,{}^{(I-4)}Z^{\gamma}{}_{\beta}\right)\Big]
\end{aligned}
$$

Hehl et al. 1999

Timeline NRC models 1972-1985



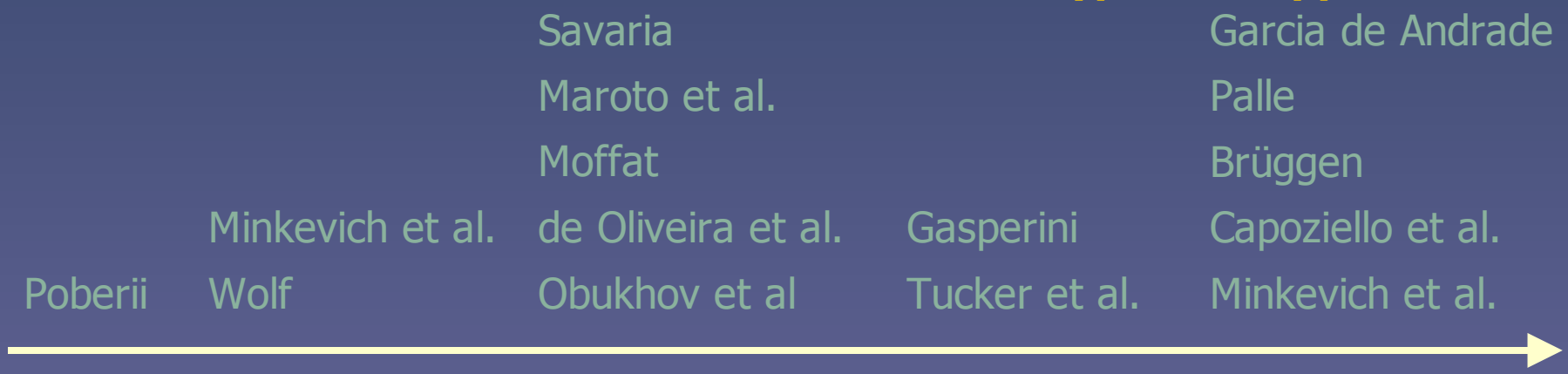
Timeline NRC models 1986-1999



Systematic study of Weyssenhoff fluid models

First Weyl & WC scenarios

Hyperfluid approach



Nonmetricity driven inflation

First model within metric-affine gravity

Timeline NRC models 2001-2004

Models enter quantitative regime

First SNIa parameter estimates in WC model

NGT model

Extended WC model

2001	2002	2003	2004
	Minkevich		Szydlowski
	Puetzfeld	Minkevich	Böhmer
	Shapiro	Vereshchagin	Scholz
Moffat	Capoziello	Babourova et al.	Moffat
Puetzfeld et al.	Puetzfeld	Capoziello et al.	Miritzis
			Puetzfeld et al.

Summary NRC

- Field equations managable for cosmological models which make use of a constrained Lagrangian
- Possible to construct viable models (passed several cosmological tests: SN Ia, X-ray cluster, FR IIb radio galaxies, BBN)
- Interesting correspondences between NRC, anisotropic, and braneworld models
- So far **no** solution of the dark energy/matter problem

Outlook NRC

Near future

- Sophisticated fluid models offer a good framework for cosmology
- Incorporate the full hyperfluid symmetries

→ **Continue search for genuine effects**

Far future

- Investigate full MAG Lagrangian (difficult!)
- Work out perturbed field equations

"...the question whether this [spacetime] continuum is Euclidean or structured according to the Riemannian scheme or **still otherwise** is a genuine physical question which has to be answered by experience rather than being a mere convention to be chosen on the basis of expediency."

A. Einstein, Geometrie und Erfahrung, translation by F.W. Hehl

„From a fundamental viewpoint it is totally wrong to aim at basing a theory only on observable quantities. For in reality it is just the other way around. Only the theory decides about what can be observed."

„Aber vom prinzipiellen Standpunkt aus ist es ganz falsch, eine Theorie nur auf beobachtbaren Größen gründen zu wollen. Denn es ist ja in Wirklichkeit genau umgekehrt. Erst die Theorie entscheidet darüber, was man beobachten kann."

Einstein according to Heisenberg (1979)

"It seems to me…that it is not so much the linearity or non-linearity which forms the heart of the matter, but the very fact that here a **more general group** than the Lorentz group is present…."

W.Pauli, Helv. Phys . Acta Suppl. 4 (1955) 261-267, translation by C. Kiefer