Status of non-Riemannian cosmology

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Overview

- Why go beyond Riemannian gravity?
- Metric-affine gravity (MAG) & non-Riemannian spacetimes
- The general MAG Lagrangian
- Short history of non-Riemannian cosmology
- A cosmological model in Weyl-Cartan spacetime
- Magnitude-redshift relation & SN Ia data
- Deceleration parameter
- Conclusion & Outlook
- References

Dark Matter 2004, Marina del Rey, Los Angeles, 17. - 20. Feb. 2004

Why go beyond Riemannian gravity?

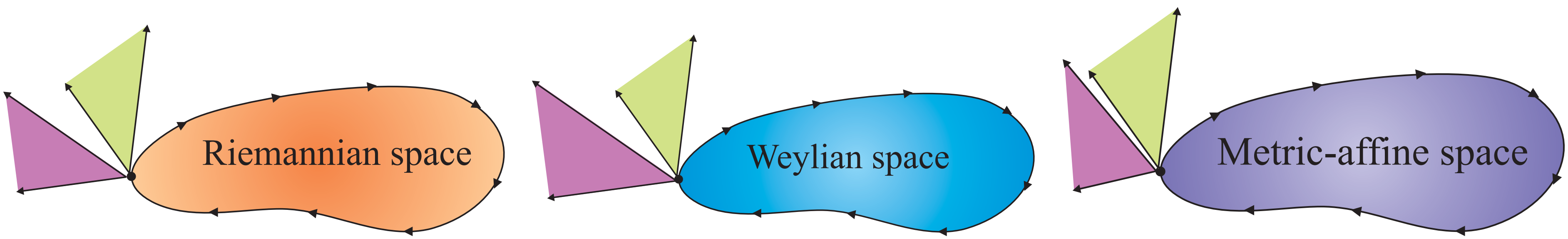
- Curved spacetime (GR) $\leftrightarrow$ dynamics of masspoints and light
- Intrinsic properties of particles suggest coupling to new fields
- Simple analogy: flat spacetime (SR) $\rightarrow$ curved spacetime (GR) $\rightarrow$ why not more complex?
- So far no unification of GR with other fundamental interactions

Why cosmology?

- Local tests in rather good compliance with GR (maybe not on very short distances)
- New effects likely to show up on large scales and at high energies
- Cosmology entered a *golden age* $\rightarrow$ observational situation very promising
- Observations force us to introduce concepts like dark matter and dark energy
- So far *no* direct detection of a dark matter particle

"... the question whether this [spacetime] continuum is Euclidean or structured according to the Riemannian scheme or still otherwise is a genuine physical question which has to be answered by experience rather than being a mere convention to be chosen on the basis of expediency."

A. Einstein: *Geometrie und Erfahrung*, translation by F.W. Hehl



$$\Gamma_{\alpha\beta} = \widetilde{\Gamma}_{\alpha\beta} + N_{\alpha\beta} = \underbrace{\frac{1}{2}dg_{\alpha\beta} + \left(e_{[\alpha}\rfloor dg_{\alpha]\gamma}\right)\vartheta^{\gamma} + e_{[\alpha}\rfloor C_{\beta]} - \frac{1}{2}\left(e_{\alpha}\rfloor e_{\beta}\rfloor C_{\gamma}\right)\vartheta^{\gamma}}_{\text{Levi-Civita connection}}$$

$$\underbrace{-e_{[\alpha}\rfloor T_{\beta]} + \frac{1}{2}\left(e_{\alpha}\rfloor e_{\beta}\rfloor T_{\gamma}\right)\vartheta^{\gamma} + \frac{1}{2}Q_{\alpha\beta} + \left(e_{[\alpha}\rfloor Q_{\beta]\gamma}\right)\vartheta^{\gamma}}_{\text{Distortion}}$$

Metric-affine gravity (MAG)

Potentials	Field strengths	Excitations	Gauge currents	Matter currents
$g_{\alpha\beta}$	$Q_{\alpha\beta}$	$M^{\alpha\beta}$	$m^{\alpha\beta}$	$\sigma^{\alpha\beta}$
ϑ^α	T^α	H_α	E_α	Σ_α
$\Gamma_\alpha{}^\beta$	$R^\alpha{}_\beta$	$H^\alpha{}_\beta$	$E^\alpha{}_\beta$	$\Delta^\alpha{}_\beta$

Field equations	MAG	GR
0	$DM^{\alpha\beta} - m^{\alpha\beta} = \sigma^{\alpha\beta}$	$\eta_{\alpha\beta\gamma} \wedge \widetilde{R}^{\beta\gamma} + 2\lambda\eta_\alpha = 2\kappa\Sigma_\alpha$
I	$DH_\alpha - E_\alpha = \Sigma_\alpha$	
II	$DH^\alpha{}_\beta - E^\alpha{}_\beta = \Delta^\alpha{}_\beta$	

Lagrangian $L = V_{\text{MAG}} + L_{\text{mat}} = V_{\text{MAG}}(g_{\alpha\beta}, \vartheta^\alpha, Q_{\alpha\beta}, T^\alpha, R_\alpha{}^\beta) + L_{\text{mat}}(g_{\alpha\beta}, \vartheta^\alpha, \Psi, D\Psi)$

General MAG Lagrangian

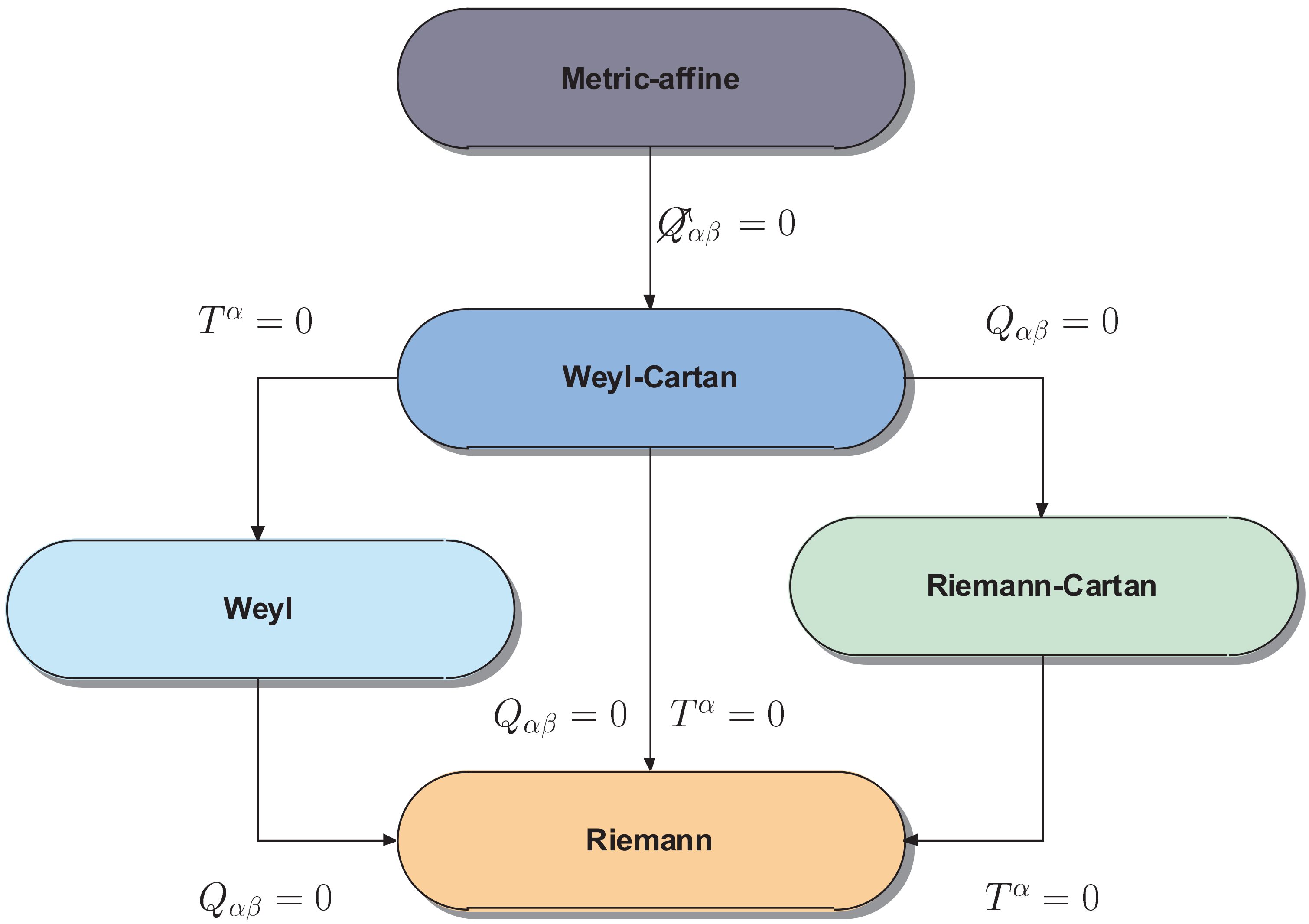
$$
\begin{aligned}
V_{\mathrm{MAG}} = {} & \frac{1}{2\kappa}\Big[-a_0 R^{\alpha\beta}\wedge\eta_{\alpha\beta} - 2\lambda\eta + T^\alpha\wedge{}^\star\Big(\sum_{I=1}^{3} a_I\,{}^{(I)}T_\alpha\Big) \\
& + Q_{\alpha\beta}\wedge{}^\star\Big(\sum_{I=1}^{4} b_I\,{}^{(I)}Q^{\alpha\beta}\Big) \\
& + b_5\Big({}^{(3)}Q_{\alpha\gamma}\wedge\vartheta^\alpha\Big)\wedge{}^\star\Big({}^{(4)}Q^{\beta\gamma}\wedge\vartheta_\beta\Big) \\
& + 2\Big(\sum_{I=2}^{4} c_I\,{}^{(I)}Q_{\alpha\beta}\Big)\wedge\vartheta^\alpha\wedge{}^\star T^\beta\Big] \\
-\frac{1}{2\rho}R^{\alpha\beta}\wedge{}^\star\Big[& \sum_{I=1}^{6} w_I\,{}^{(I)}W_{\alpha\beta} + \sum_{I=1}^{5} z_I\,{}^{(I)}Z_{\alpha\beta} + w_7\vartheta_\alpha\wedge\Big(e_\gamma\rfloor\,{}^{(5)}W^\gamma{}_\beta\Big) \\
& + z_6\vartheta_\gamma\wedge\Big(e_\alpha\rfloor\,{}^{(2)}Z^\gamma{}_\beta\Big) + \sum_{I=7}^{9} z_I\vartheta_\alpha\wedge\Big(e_\gamma\rfloor\,{}^{(I-4)}Z^\gamma{}_\beta\Big)\Big]
\end{aligned}
$$

HEHL ET AL. (1999)

Coupling constants $a_0,\ldots,a_3$ $b_1,\ldots,b_5$ $c_2,\ldots,c_4$ $w_1,\ldots,w_7$ $z_1,\ldots,z_9$

κ ρ λ

Spacetime types



Short history of Non-Riemannian Cosmology (NRC)

1980	*Minkevich*	Generalized FLRW solution in Einstein-Cartan theory, avoids singularity
1983	*Minkevich*	Early de Sitter type solutions in a theory with torsion
1990	*Kao*	Inflationary solutions in a Weyl invariant scenario
1994	*Poberii*	Inflationary solutions driven by nonmetricity
1993	*Tresguerres*	First cosmological model in Weyl-Cartan spacetime
1995	*Minkevich*	Qualitative analysis of an exact solution in Weyl-Cartan spacetime
1996	*Hammond*	Helicity flip crossection for a theory with torsion
1997	*Obukhov et al.*	First cosmological model within the triplet ansatz of MAG
1998	*Tucker & Wang*	Exact solutions within a triplet model, qualitative discussion of a new DM coupling related to the Proca charge of particles
1998	*Gasperini*	Avoidance of the initial singularity due to a spin-torsion interaction
1999	*Brüggen*	Possible effects of torsion on BBN (based on work of Hammond)
1999	*Palle*	Density fluctuations in Einstein-Cartan theory
2001	*Puetzfeld & Tresguerres*	Exact solutions and qualitative analysis of a model in Weyl-Cartan spacetime
2002	*Puetzfeld*	Exact solutions of an new model in Weyl-Cartan spacetime
2002	*Puetzfeld*	Parameter constraints from SN Ia data within the Weyl-Cartan model
2002	*Barbourova & Frolov*	Derivation of a model in Weyl-Cartan spacetimes (partially resembles the model of Obukhov et al. and the model of Puetzfeld)
2004	*Puetzfeld & Chen*	Comprehensive analysis of the SN Ia data within a Weyl-Cartan model

Cosmological model in Weyl-Cartan spacetime

Lagrangian $V = \frac{\chi}{2\kappa} R_{\alpha}{}^{\beta} \wedge \eta^{\alpha}{}_{\beta} + \sum_{I=1}^{6} a_I \, R_{\alpha}{}^{\beta} \wedge \, {}^{\star(I)}W^{\alpha}{}_{\beta} + b \, R_{\alpha}{}^{\beta} \wedge \, {}^{\star(4)}Z^{\alpha}{}_{\beta} + c \, Q_{\alpha\beta} \wedge \, {}^{\star(4)}Q^{\alpha\beta}$

$\sim$ Einstein-Hilbert + quadratic rotational curvature
+ quadratic strain curvature + quadratic nonmetricity

Puetzfeld (2002a,2002b)

Assumptions $Q_{\alpha\beta} = g_{\alpha\beta} Q \sim {}^{(4)}Q_{\alpha\beta}$ $\quad T^{\alpha} = \frac{1}{2} Q \wedge \vartheta^{\alpha} \sim {}^{(2)}T^{\alpha}$

Σ_{α} ideal fluid $\not{\Delta}_{(\alpha\beta)} = \tau_{\alpha\beta} = 0$ vanishing shear & spin $\Delta_{\alpha\beta} = \Delta \backsim Q$ purely dilational

$$\vartheta^{\hat{0}} = dt \qquad \vartheta^{\hat{1}} = \frac{S(t)}{\sqrt{1-kr^2}} dr \qquad \vartheta^{\hat{2}} = S(t)\, r \, d\theta \qquad \vartheta^{\hat{3}} = S(t)\, r \, \sin\theta d\phi$$

$$ds^2 = \vartheta^{\hat{0}} \otimes \vartheta^{\hat{0}} - \vartheta^{\hat{1}} \otimes \vartheta^{\hat{1}} - \vartheta^{\hat{2}} \otimes \vartheta^{\hat{2}} - \vartheta^{\hat{3}} \otimes \vartheta^{\hat{3}}$$

Parameters $\chi \quad a_{I=1\ldots6} \quad b \quad c \quad k = -1, 0, 1$

Equation of state $p = w\,\mu$

Functions $\mu(t) \quad p(t) \quad S(t)$

Make ansatz for Q → derive field eqs.

Magnitude-redshift relation (special case)

Weyl 1-form

$$Q = -\frac{\kappa\psi}{8b_4} S^{-3} dt \qquad \psi,\, b_4,\, \kappa = \text{const} \qquad \upsilon := \frac{\kappa^2}{144 a_0}\left(1 - \frac{3a_0}{b_4}\right) = \text{const}$$

Field equations

$$\frac{\dot{S}^2}{S^2} + \frac{k}{S^2} - \frac{\lambda}{3} = \frac{\kappa}{3}\left[\mu + \frac{\kappa}{48 a_0}\left(1 - \frac{3a_0}{b_4}\right)\frac{\psi^2}{S^6}\right]$$

$$2\frac{\ddot{S}}{S} + \frac{\dot{S}^2}{S^2} + \frac{k}{S^2} - \lambda = -\kappa\left[p + \frac{\kappa}{48 a_0}\left(1 - \frac{3a_0}{b_4}\right)\frac{\psi^2}{S^6}\right]$$

Density parameters

$$\Omega_w = \frac{\kappa}{3H^2}\mu \qquad \Omega_\lambda = \frac{\lambda}{3H^2} \qquad \Omega_\psi = \upsilon\frac{\psi^2}{S^6 H^2} \qquad \Omega_k = -\frac{k}{S^2 H^2}$$

I. Field equation

$$1 + \frac{k}{S^2H^2} - \frac{\lambda}{3} = \frac{\kappa}{3H^2}\mu + \upsilon\frac{\psi^2}{S^6H^2} \quad \Rightarrow \quad \Omega_k + \Omega_\lambda + \Omega_w + \Omega_\psi = 1$$

Hubble rate

$$H^2 = H_0^2\left[\Omega_{w0}(1+z)^{3(1+w)} + \Omega_{k0}(1+z)^2 + \Omega_{\lambda 0} + \Omega_{\psi 0}(1+z)^6\right]$$

$$d_{\text{luminosity}} = \frac{(1+z)}{H_0\sqrt{|1-\Omega_{w0}-\Omega_{\lambda 0}-\Omega_{\psi 0}|}}\,\Theta\left[\sqrt{|1-\Omega_{w0}-\Omega_{\lambda 0}-\Omega_{\psi 0}|}\int_0^z \{H_0/H(\tilde{z}\,|\,\Omega_{w0},\Omega_{\lambda 0},\Omega_{\psi 0})\}\,d\tilde{z}\right]$$

Magnitude-redshift rel.

$$m(z\,|\,H_0,\Omega_{w0},\Omega_{\lambda 0},\Omega_{\psi 0},w,M) := M + 5\log\left(\frac{d_{\text{luminosity}}}{\text{length}}\right) + 25 = M + \mu$$

Datasets

Symbol	# SN	Reference	Comments
I	18	p. 571, [121]	Calán/Tololo survey
II	42	p. 570, [121]	Supernova Cosmology Project
III	10	p. 1021, [126]	High-z Supernova Search Team
IV	10	p. 1020, [126]	Same as III but MLCS method
V	1	[125]	Farthest SN Ia observed to date
VI	27	p. 1035, [126]	Low-redshift MLCS/template
VII	230	pp.33-40, [144]	Most recent compilation of SN Ia
VIII	9	[145]	IfA Deep Survey

Combined data set of Wang contains **92** SN Ia (i.e. I+II+IV+VI−outliers)

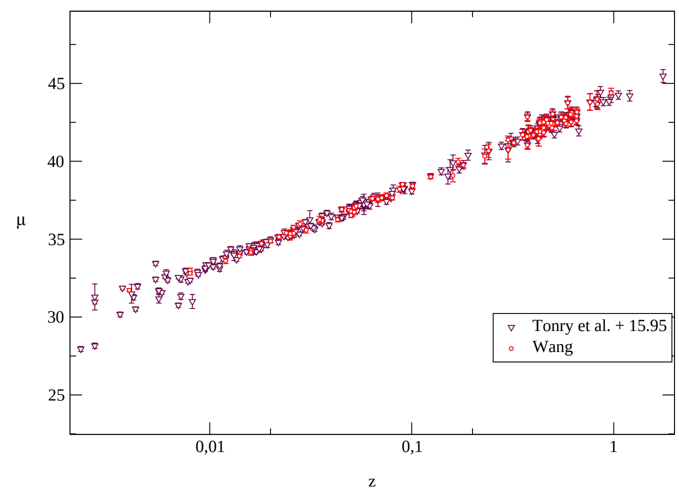
Data set of Tonry et al. contains **230** SN Ia

Fit-function

$$\chi^2 := \sum_{i=1}^{92/230} \frac{\left[\mu_i^{\text{theory}}(z_i|\text{parameters}) - \mu_i^{\text{measured}}\right]^2}{\sigma_{\mu\ i}^2 + \sigma_{mz\ i}^2}$$

Error

$$\sigma_{mz} := \frac{5}{ln10}\left[\frac{1}{d_{\text{luminosity}}}\frac{\partial d_{\text{luminosity}}}{\partial z}\right]\sigma_z$$



Unbinned data sets

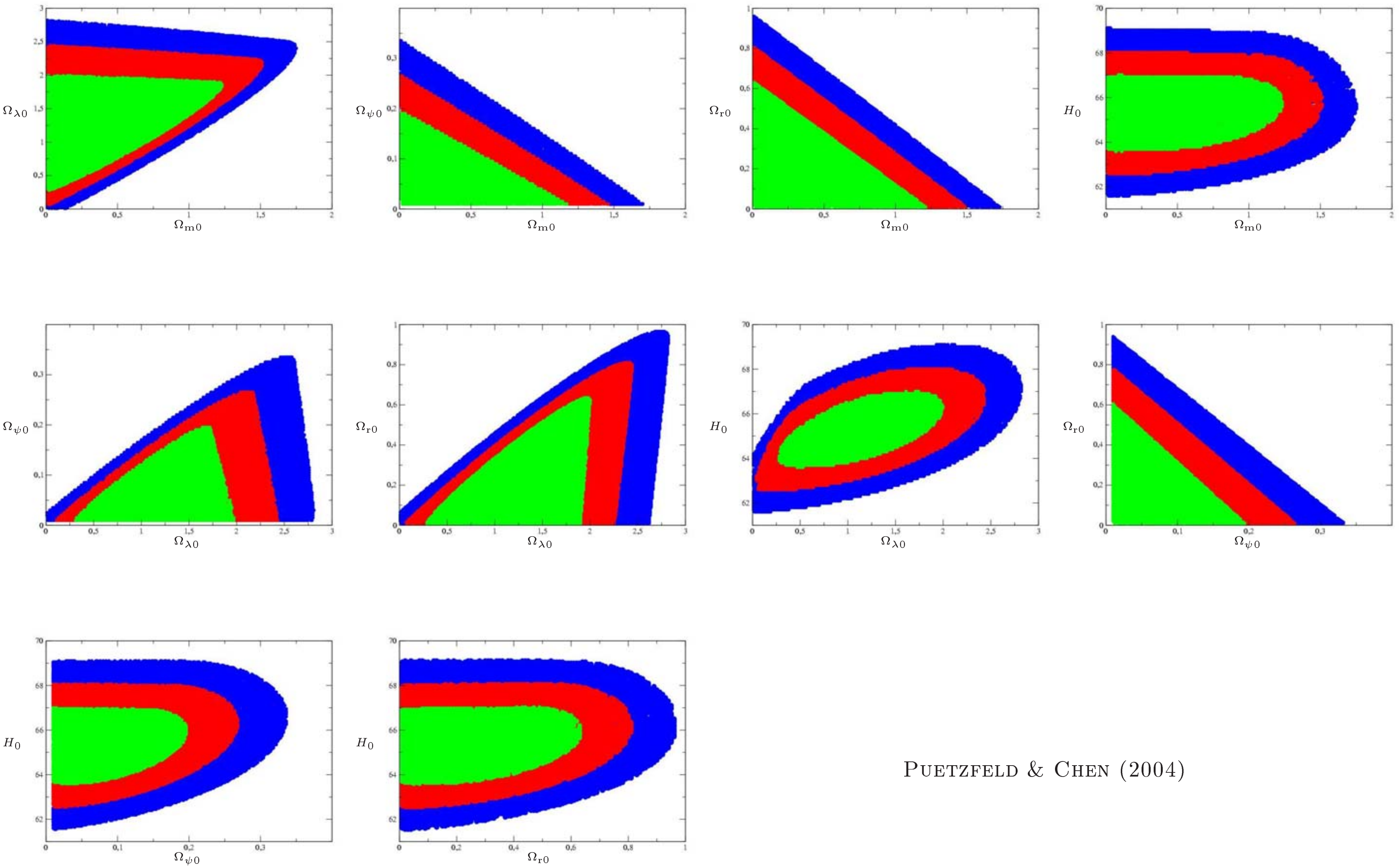
Data set	$\sharp$ **SN**	$\Omega_{\mathrm{m}0}$	$\Omega_{\lambda 0}$	$\Omega_{\psi 0}$	$\Omega_{\mathrm{r}0}$	H_0	χ^2	χ^2_ν	q_0
W92	92	0.076	1.218	0.001	0.295	65.334	134.446	1.54	-0.88
W230	230	0.013	1.446	0.001	0.363	66.300	249.112	1.11	-1.07

Binned data sets

Data set	$\sharp$ **SN**	Δz	$\Omega_{\mathrm{m}0}$	$\Omega_{\lambda 0}$	$\Omega_{\psi 0}$	$\Omega_{\mathrm{r}0}$	H_0	χ^2	χ^2_ν	q_0
W114	114	0.002	0.001	1.986	0.155	0.165	67.861	98.620	0.90	-1.51
W25	25	0.04	0.860	6.567	0.754	0.270	75.663	29.954	1.49	-4.35

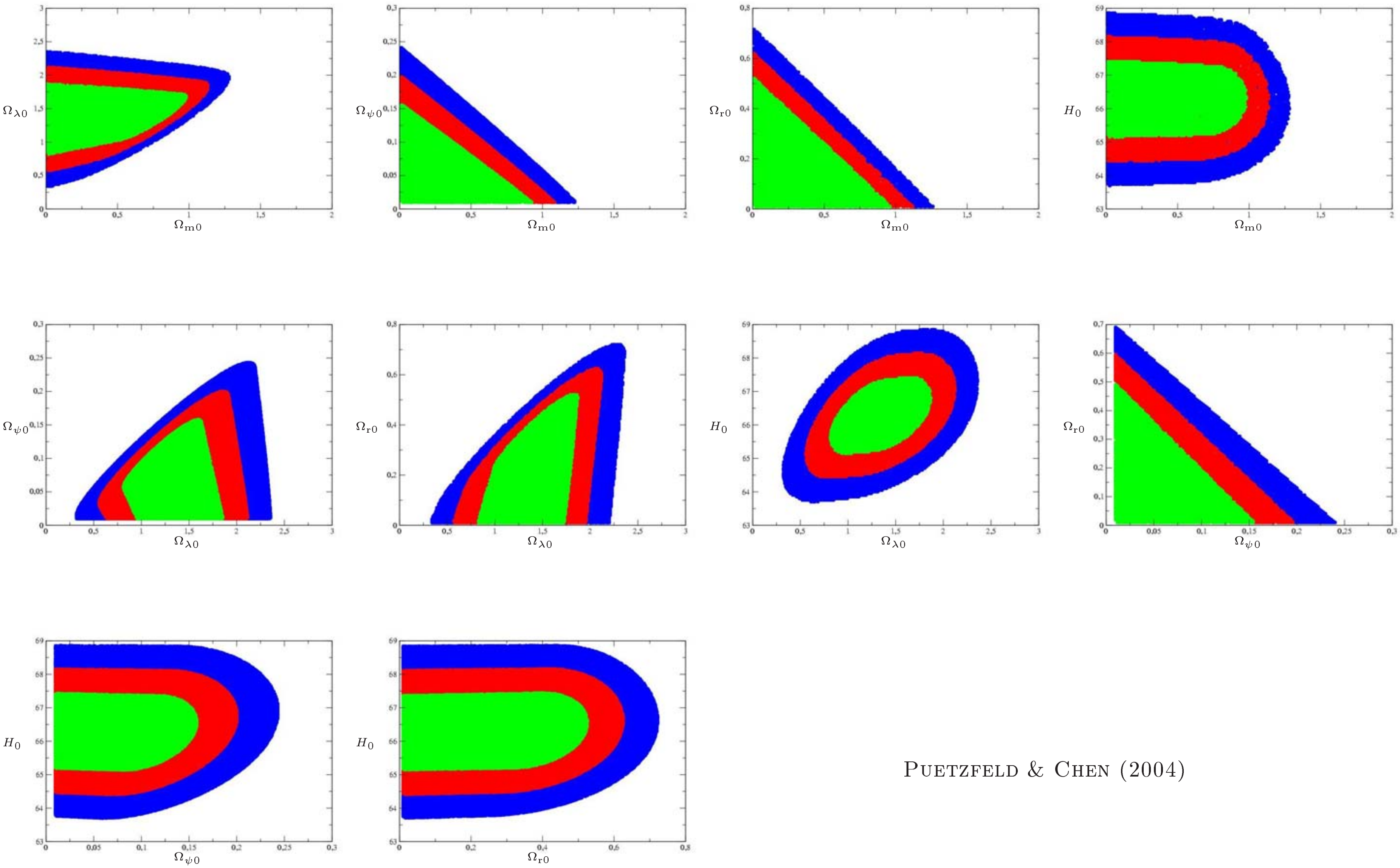
Puetzfeld & Chen (2004)

Confidence contours (unbinned data set, 92 SNe Ia)



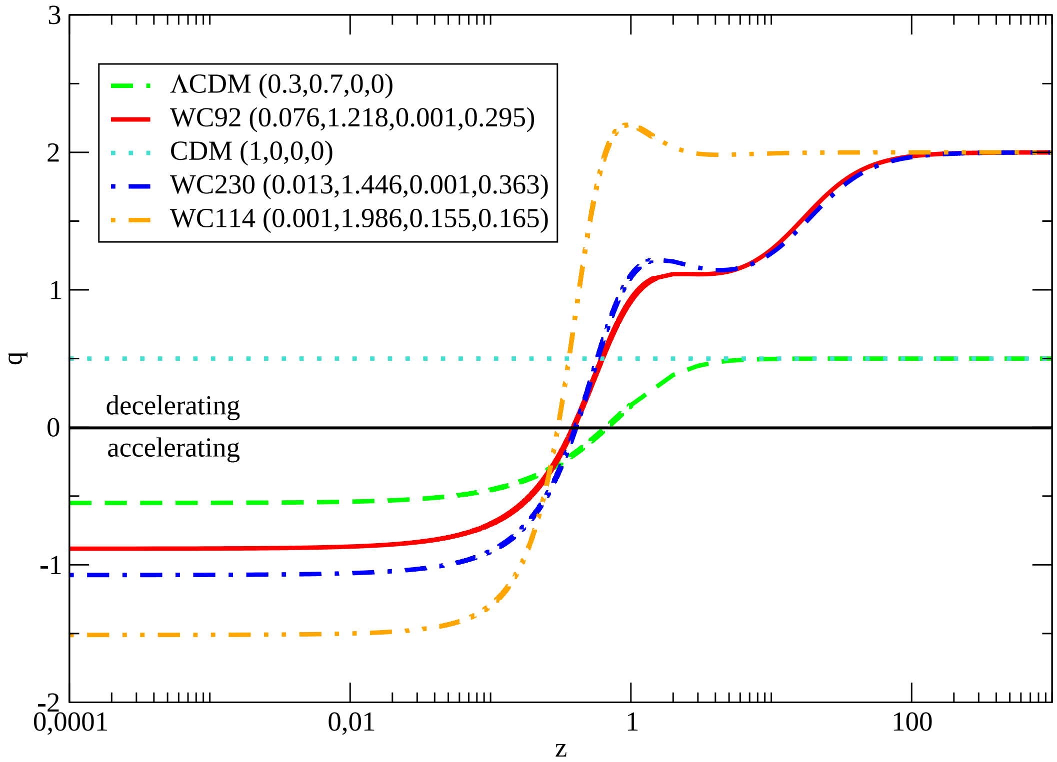
PUETZFELD & CHEN (2004)

Confidence contours (unbinned data set, 230 SNe Ia)



PUETZFELD & CHEN (2004)

Deceleration parameter



$$q = \frac{\Omega_{\mathrm{m}0}(1+z)^3 - 2\Omega_{\lambda 0} + 4\Omega_{\psi 0}(1+z)^6 + 2\Omega_{\mathrm{r}0}(1+z)^4}{2\left[z\Omega_{\mathrm{m}0}(1+z)^2 - z\Omega_{\lambda 0}(2+z) + z\Omega_{\psi 0}\sum_{i=2}^{5}(1+z)^i + z\Omega_{\mathrm{r}0}\sum_{i=2}^{3}(1+z)^i + (1+z)^2\right]}$$

Conclusion & Outlook

- Exact solution in a cosmological context
- Passes two distinct cosmological tests (magnitude-redshift relation, nucleosynthesis)
- Early estimates from third test (CMB) compatible with previous results
- Parameters constrained with the help of real data sets (!)

- Model not worked out to the same level of detail as the standard cosmological model
- Many ad-hoc assumptions $\rightarrow$ viable but no real predictive power
- Model still far away from full MAG scenario

- Cosmological tests should help us to find the correct MAG Lagrangian
- Ideal starting point in order to derive existing phenomenological models from a Lagrangian
- Description of SN Ia data without cosmological constant seems to be possible with slight modifications
- May shed light on the dark matter problem via a modification of the dynamics on galactic scales

References

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