A cosmological model in Weyl-Cartan spacetime

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OVERVIEW

- Metric-affine gravity (MAG)
- Weyl-Cartan spacetime
- Lagrangian & Field equations
- Solutions
- Conclusion & Outlook

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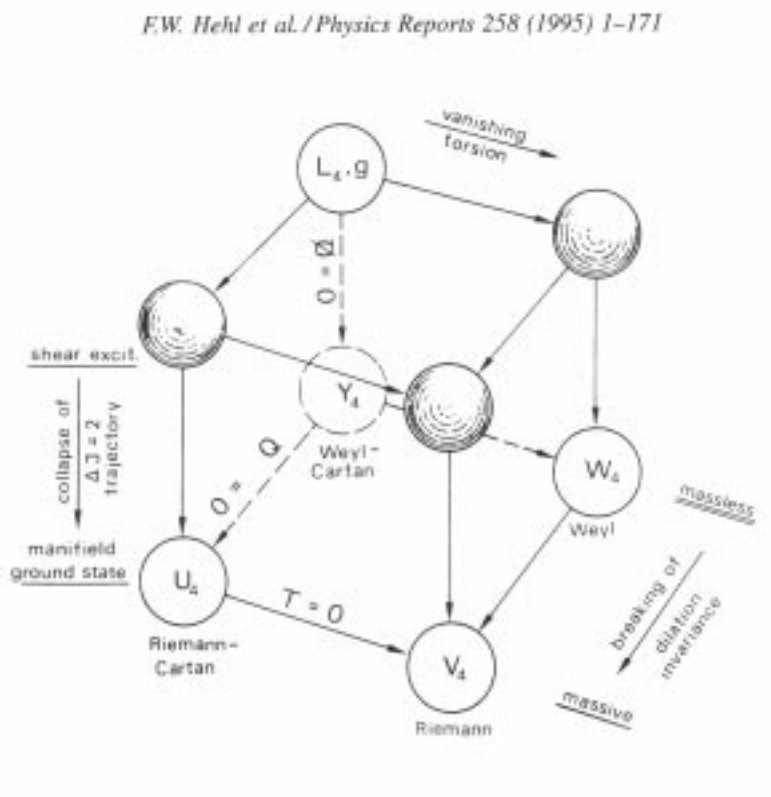
Metric-affine gravity (MAG)

Potentials	Field strengths	Excitations	Gauge currents	Matter currents
$g_{\alpha\beta}$	$Q_{\alpha\beta}$	$M^{\alpha\beta}$	$m^{\alpha\beta}$	$\sigma^{\alpha\beta}$
ϑ^{α}	T^{α}	H_{α}	E_{α}	Σ_{α}
$\Gamma_{\alpha}{}^{\beta}$	$R^{\alpha}{}_{\beta}$	$H^{\alpha}{}_{\beta}$	$E^{\alpha}{}_{\beta}$	$\Delta^{\alpha}{}_{\beta}$

Field equations	MAG	GR
I	$DH_{\alpha} - E_{\alpha} = \Sigma_{\alpha}$	$\eta_{\alpha\beta\gamma} \wedge \tilde{R}{}^{\beta\gamma} + 2\Lambda\eta_{\alpha} = 2\kappa\Sigma_{\alpha}$
II	$DH^{\alpha}{}_{\beta} - E^{\alpha}{}_{\beta} = \Delta^{\alpha}{}_{\beta}$	

Lagrangian $L = V_{\mathrm{MAG}} + L_{\mathrm{mat}} = V_{\mathrm{MAG}}(g_{\alpha\beta}, \vartheta^{\alpha}, Q_{\alpha\beta}, T^{\alpha}, R_{\alpha}{}^{\beta}) + L_{\mathrm{mat}}(g_{\alpha\beta}, \vartheta^{\alpha}, \Psi, D\Psi)$

Weyl-Cartan spacetime (Y_n)



Properties

Weyl 1-form $\quad Q := \frac{1}{4} Q^{\alpha}{}_{\alpha}$

$$Q_{\alpha\beta} \quad \backsim \quad Z_{\alpha\beta} \quad \backsim \quad Q$$

Hypermomentum current

$$\begin{aligned} \Delta_{\alpha\beta} &= \text{antisymmetric piece} + \text{trace piece} \\ &= \tau_{\alpha\beta} + \frac{1}{4} g_{\alpha\beta}\, \Delta = \tau_{\alpha\beta} + \frac{1}{4} g_{\alpha\beta}\, \Delta^{\gamma}{}_{\gamma} \\ &= \text{spin current} + \text{dilation current} \end{aligned}$$

Second MAG field equation

$$\begin{aligned} dH^{\alpha}{}_{\alpha} - E^{\alpha}{}_{\alpha} &= \Delta \\ g_{\gamma[\alpha} DH^{\gamma}{}_{\beta]} - E_{[\alpha\beta]} &= \tau_{\alpha\beta} \end{aligned}$$

Lagrangian & Field equations

Lagrangian $V = \frac{\chi}{2\kappa} R_\alpha{}^\beta \wedge \eta_\beta{}^\alpha + \sum_{I=1}^{6} a_I \, {}^{(I)}W_\alpha{}^\beta \wedge {}^\star R_\beta{}^\alpha + b \, Z_{\alpha\beta} \wedge {}^\star R^{\beta\alpha}$

$=$ Einstein-Hilbert $+$ quadratic rotational curvature $+$ quadratic strain curvature

Assumptions $\tau_{\alpha\beta} = 0$ $\quad \Sigma_\alpha$ ideal fluid $\quad T^\alpha \quad \backsim \quad \Delta \quad \backsim \quad Q$

$$\vartheta^{\hat{0}} = dt \qquad \vartheta^{\hat{1}} = \frac{S(t)}{\sqrt{1-kr^2}} dr \qquad \vartheta^{\hat{2}} = S(t)\, r\, d\theta \qquad \vartheta^{\hat{3}} = S(t)\, r\, \sin\theta d\phi$$

$$ds^2 = \vartheta^{\hat{0}} \otimes \vartheta^{\hat{0}} - \vartheta^{\hat{1}} \otimes \vartheta^{\hat{1}} - \vartheta^{\hat{2}} \otimes \vartheta^{\hat{2}} - \vartheta^{\hat{3}} \otimes \vartheta^{\hat{3}}$$

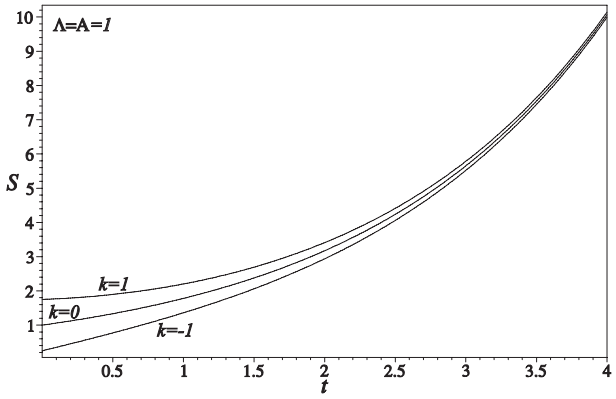
Field equations $\frac{\ddot{S}}{S} + \left(\frac{\dot{S}}{S}\right)^2 + \frac{k}{S^2} = \frac{2}{3}\Lambda$

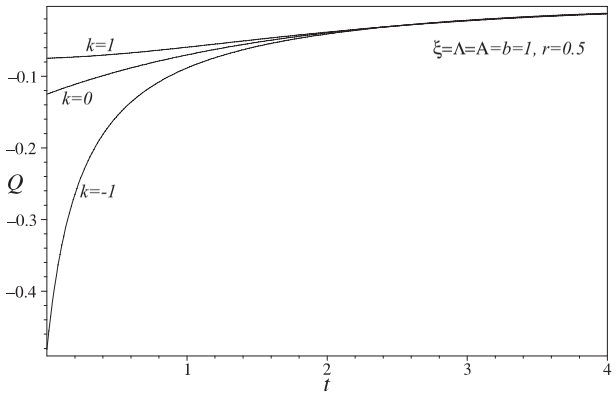
$$\chi\Lambda = 0$$

$$\chi\frac{\ddot{S}}{S} + \frac{4\kappa\Lambda}{3}(a_4 + a_6)\left(\frac{\ddot{S}}{S} - \frac{\Lambda}{3}\right) = -\frac{\kappa\mu_0}{3S^4}$$

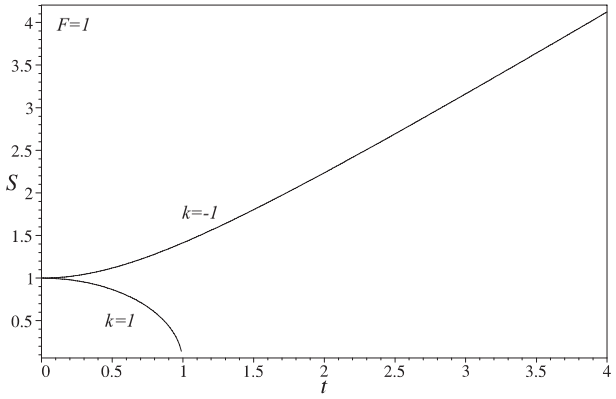
Solutions

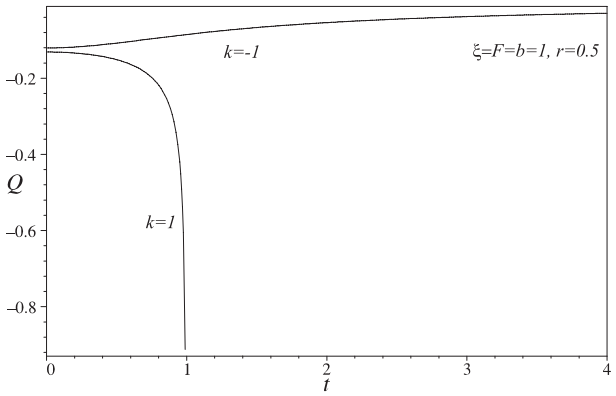
Vacuum solution



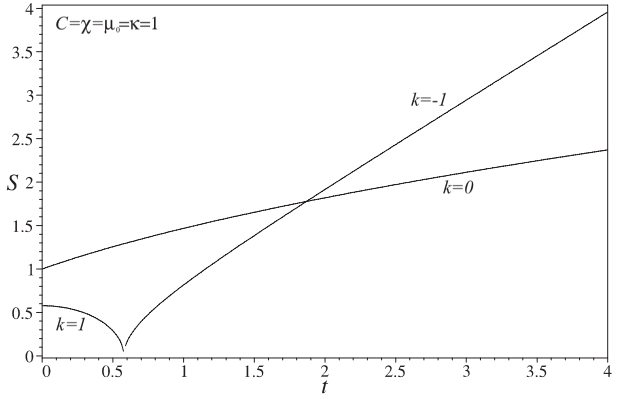


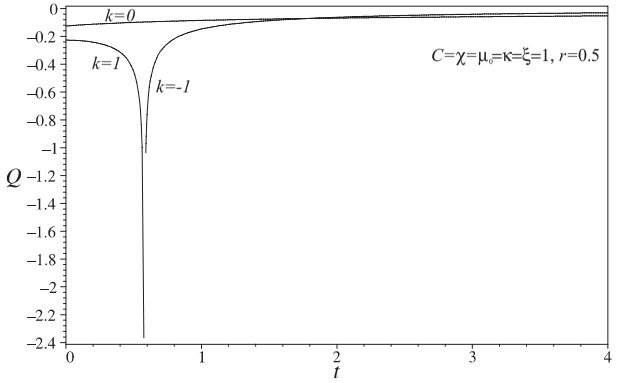
Intermediate solution





Radiative solution





Conclusion & Outlook

- Field equations stay at manageable size (if you make use of CA)
- Exact solutions with reasonable behavior of the scale factor
- Post-Riemannian quantities *die out* at late stages of the universe
- Model does not incorporate the full MAG symmetries
- Very restricted matter model
- Further extension of the gauge Lagrangian (e.g. additional quadratic nonmetricity terms)
- Incorporation of anisotropic metrical structures
- Modify parameters in order to fit recent observational data
- Address problems of standard cosmology
- Comparison with existing cosmological models (e.g. Inflation)

References

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