Primordial nucleosynthesis

Dirk Pützfeld

Email: dp@thp.uni-koeln.de WWW: http://www.thp.uni-koeln.de/~dp

Overview

- Motivation & The cornerstones of cosmology
- History of the universe within the standard picture
- Neutron-to-baryon ratio in an expanding universe
- Synthesis of light elements
- Deuterium abundance and capture time
- Computer programs for nucleosynthesis calculations
- Combined observations & Other groups
- Conclusion & Outlook
- References

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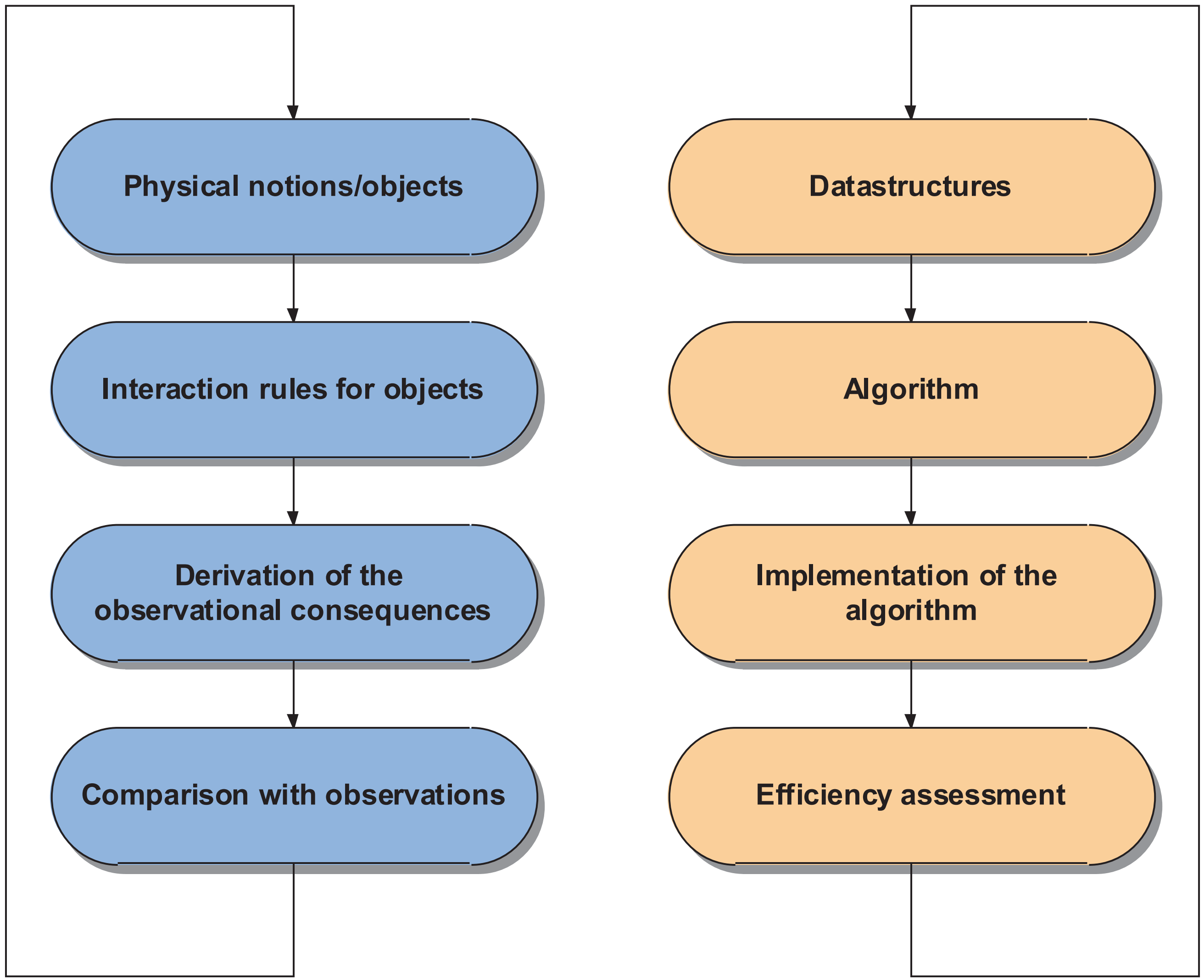
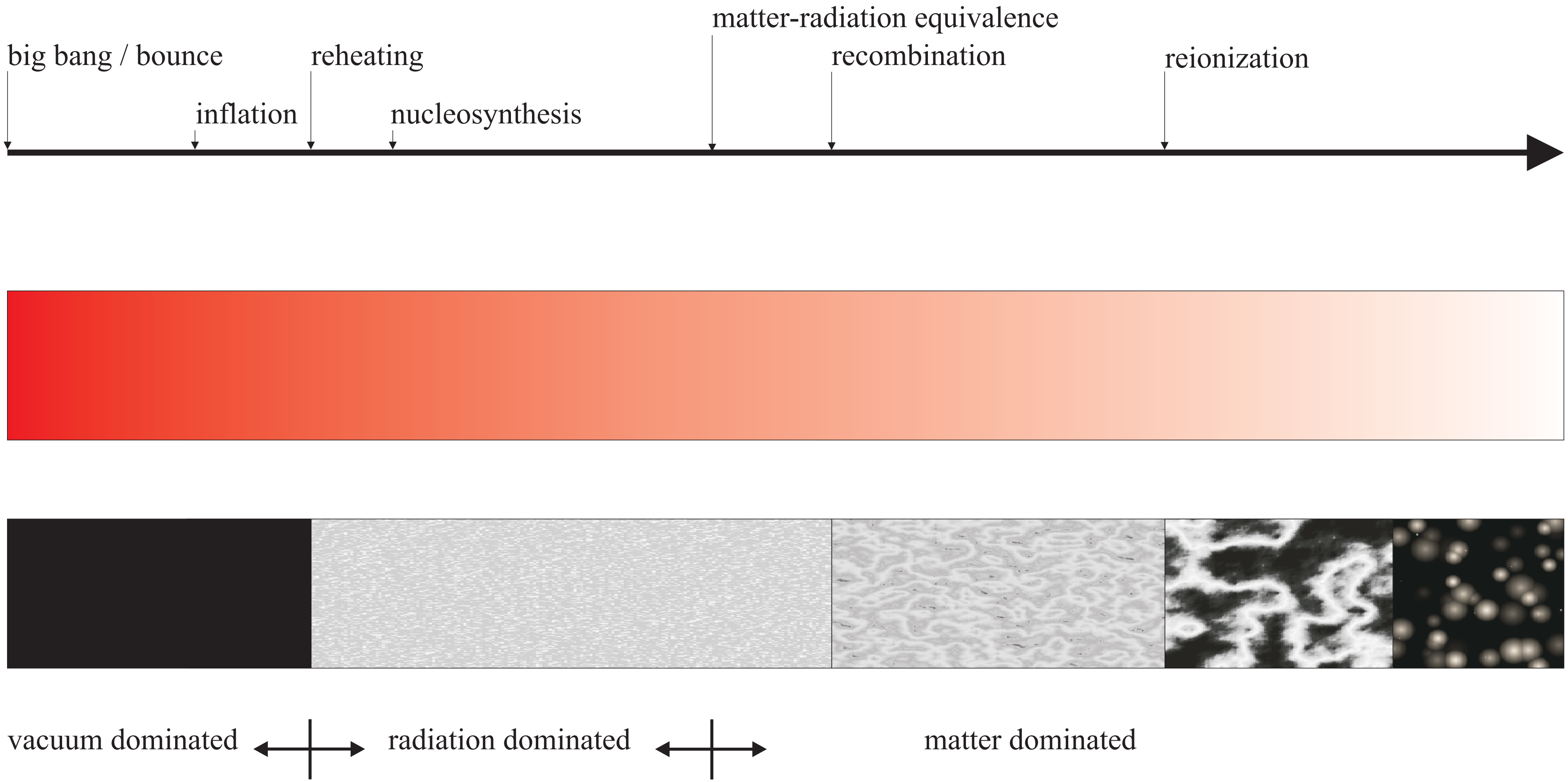
Motivation



Physical notions/objects
Interaction rules for objects
Derivation of the observational consequences
Comparison with observations
Datastructures
Algorithm
Implementation of the algorithm
Efficiency assessment

History of the universe



Neutron-to-baryon ratio

Processes

$$n + \nu_e \quad \longleftrightarrow \quad p + e^-$$

$$n + e^+ \quad \longleftrightarrow \quad p + \bar{\nu}_e$$

$$n \quad \longleftrightarrow \quad p + e^- + \bar{\nu}_e$$

$$Y_n := \frac{n_n}{n_n + n_p}$$

$$\frac{dY_n}{dt} = \Gamma_{pn} \left(1 - Y_n\right) - \Gamma_{np} Y_n$$

Reaction rates

$$\Gamma_{n\nu_e \to pe^-} = 2AT^3 \left(Q^2 + 6TQ + 12T^2\right) = \Gamma_{ne^+ \to p\bar{\nu}_e}$$

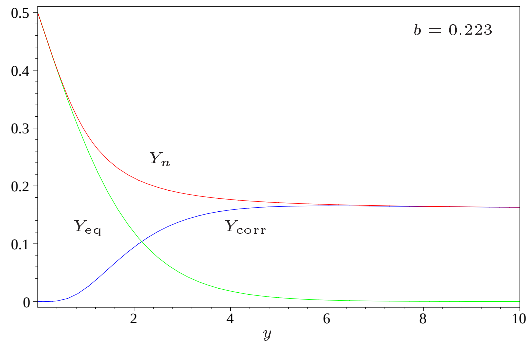
$$\Gamma_{n \to p\bar{\nu}_e e^-} = \frac{A\sqrt{Q^2 - m_e^2}}{5} \left(\frac{Q^4}{6} - \frac{3Q^2 m_e^2}{4} - \frac{2m_e^4}{3}\right) + \frac{AQm_e^4}{4} \operatorname{arccosh}\left(\frac{Q}{m_e}\right)$$

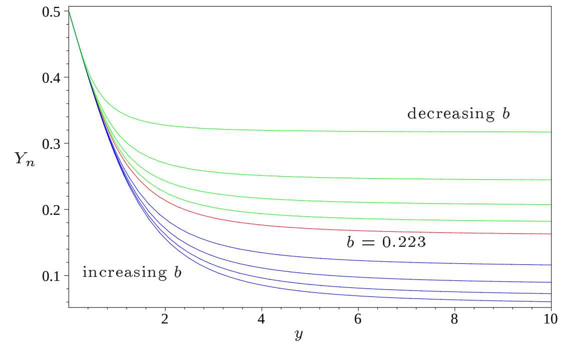
$$Y_n\left(y, Q, \tau, g_{\mathbf{total}}\right) = \frac{1}{1+e^y} + \int_0^y d\tilde{y}\ \frac{e^{\tilde{y}}}{\left(1+e^{\tilde{y}}\right)^2} \exp\left[b\left(\frac{4}{\hat{y}^3} + \frac{3}{\hat{y}^2} + \frac{1}{\hat{y}}\right) + b\ e^{-\hat{y}}\left(\frac{4}{\hat{y}^3} + \frac{1}{\hat{y}^2}\right)\Bigg|_y^{\tilde{y}}\right]$$

$$Q := m_n - m_p$$

$$b := \sqrt{\frac{90\ a^2}{Q^4 \tau^2 \pi^2 \kappa g_{\text{total}}}}$$

$$y := Q/T$$



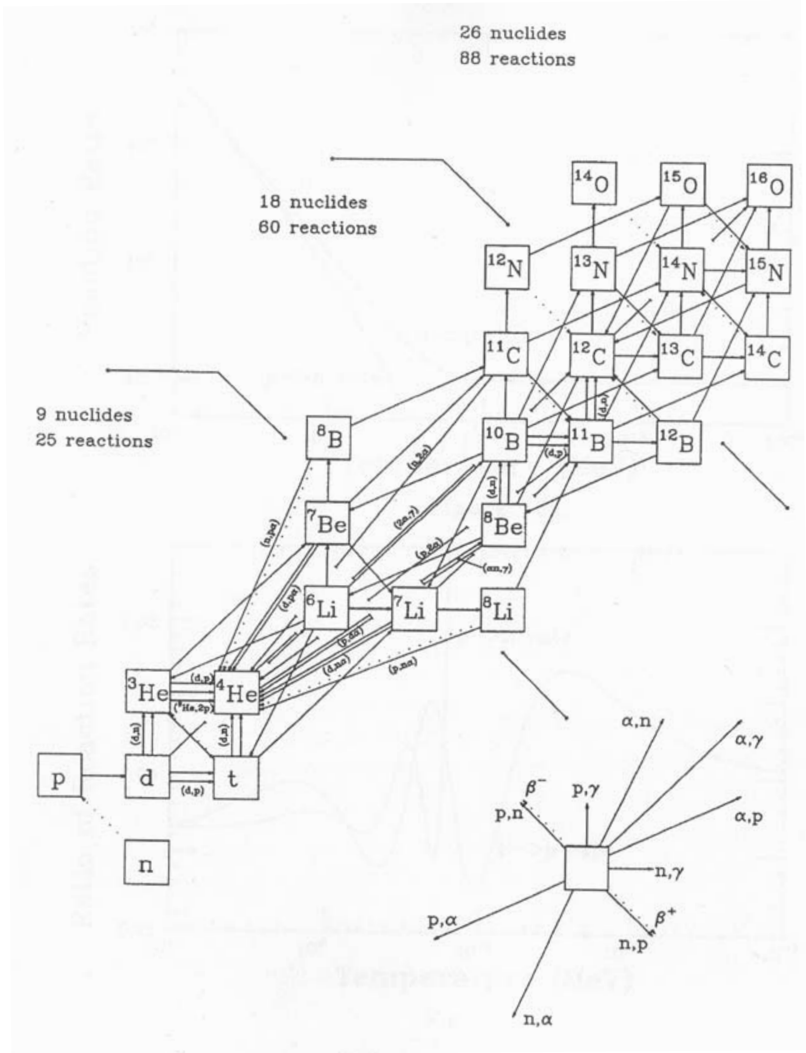


Synthesis of light elements

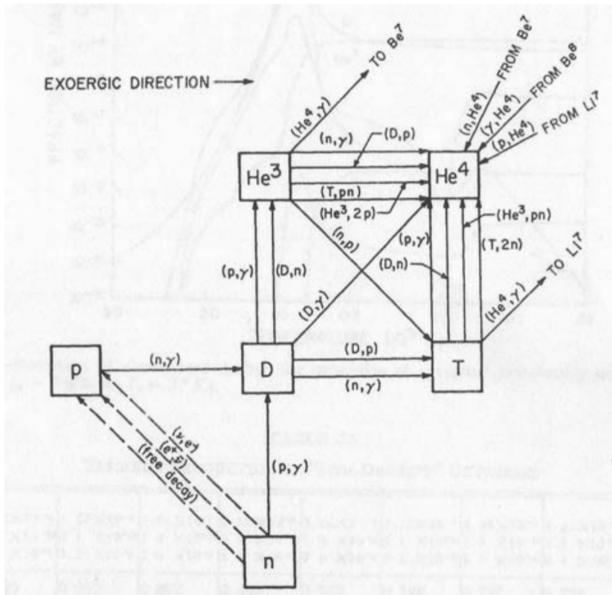
$$n + p \to \mathrm{D} + \gamma$$

$$\mathrm{D} + \mathrm{D} \to {}^{3}\mathrm{H} + p$$

$$\mathrm{D} + \mathrm{T} \to {}^{4}\mathrm{He} + n$$



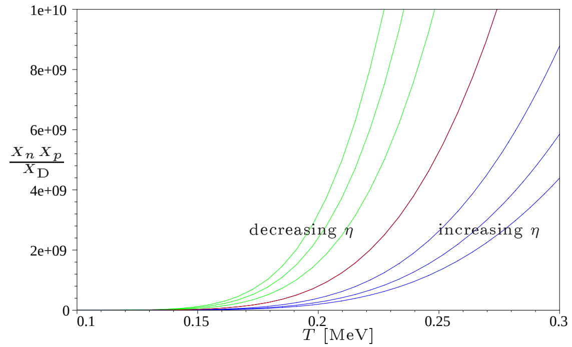
Kawano (1992)

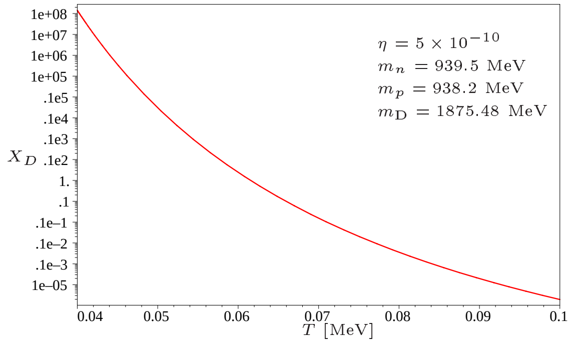


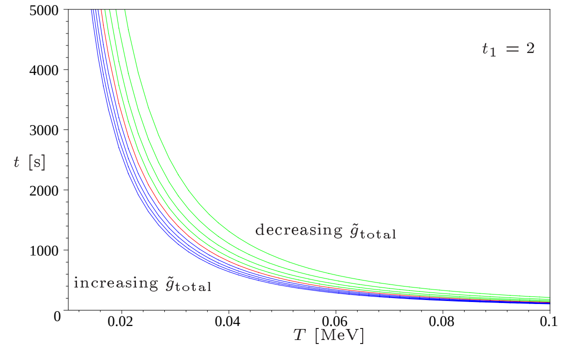
Wagoner, Fowler, Hoyle (1967)

Deuterium production and capture time

$$X_{\mathbf{D}} := \frac{n_{\mathrm{D}}}{n_{\mathrm{baryons}}} = \sqrt{\frac{18}{\pi}}\zeta\left(3\right)\left(\frac{m_n m_p}{m_{\mathbf{D}}}\right)^{-\frac{3}{2}}\eta\; e^{\frac{\varepsilon_{\mathbf{D}}}{T}}T^{\frac{3}{2}}Y_n\left(1-Y_n\right) \overset{Y_n(\infty)}{\approx} \sqrt{\frac{9}{50\;\pi}}\zeta\left(3\right)\left(\frac{m_n m_p}{m_{\mathbf{D}}}\right)^{-\frac{3}{2}}\eta\; e^{\frac{\varepsilon_{\mathbf{D}}}{T}}T^{\frac{3}{2}}$$





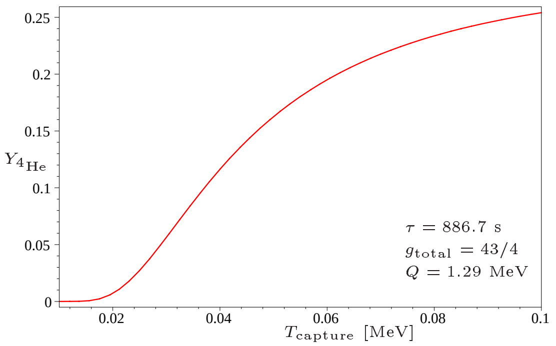


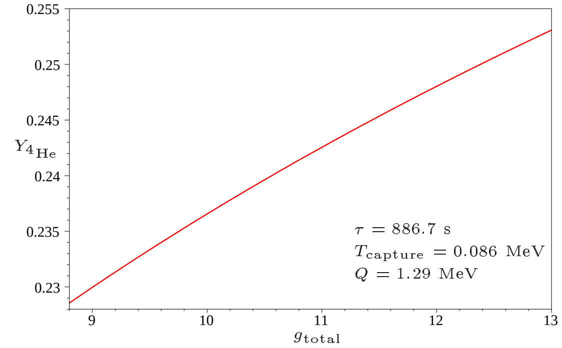
Nuclei	**Binding energy [MeV]**
^{2}H	2.22
^{3}H	6.92
^{3}He	7.72
^{4}He	28.29
^{6}Li	31.99
^{7}Li	39.24

$$t = \sqrt{\frac{45}{2\pi^2\kappa\tilde{g}_{\mathrm{total}}}}\left(\frac{11}{4}\right)^{\frac{2}{3}}T_\gamma^{-2} + 2\ \mathrm{s}$$

$$X_{\mathbf{D}} \simeq 1 \Leftrightarrow T \simeq 0.1\ \mathbf{MeV} \Leftrightarrow t \approx 133\ \mathbf{s}$$

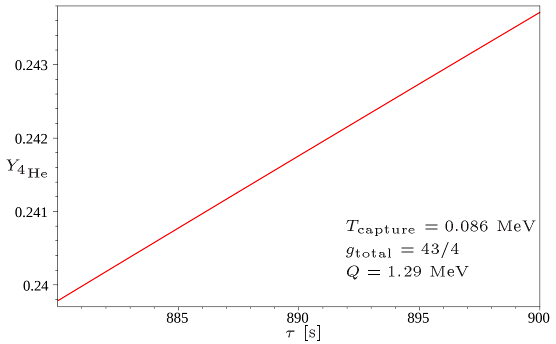
Helium-4 abundance

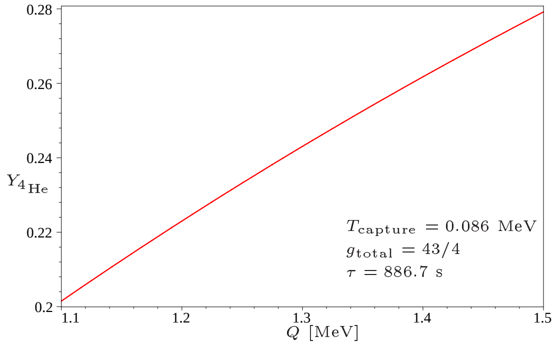




$$\breve{Y}_n := e^{-\frac{t}{\tau}} Y_n$$

$$Y_{^4\mathbf{He}} := 4\frac{n_{^4\mathbf{He}}}{n_{\mathbf{baryons}}} = \frac{2n_n}{n_p+n_n} = 2\breve{Y}_n$$





$$\tau = 886.7 \text{ s}$$
$$T_{\text{capture}} = 0.086\text{MeV}$$
$$Q = 1.29 \text{ MeV}$$
$$g_{\text{total}} = 43/4$$

$$Y_{^4\mathbf{He}} = 2\ e^{-\frac{180}{886.7}} Y_n = 2 \times 0.816 \times 0.149 = 0.243$$

Quantity	Year	Observation	Value	Ref.
$\frac{n_D}{n_H}$	1998	Local interstellar medium	$(1.5 \pm 0.1) \times 10^{-5}$	[194]
$\frac{n_D}{n_H}$	1998	Atmosphere of Jupiter	$(2.6 \pm 0.7) \times 10^{-5}$	[193]
$\frac{n_D}{n_H}$	1972	Solar system observations	$(2.5 \pm 0.5) \times 10^{-5}$	[195]
$\frac{n_D}{n_H}$	1998	Solar system observations (indirect via ^{3}He)	$(2.1 \pm 0.5) \times 10^{-5}$	[192]
$\frac{n_D}{n_H}$	1996	Ly-α observations Q1937-1009 ($z = 3.572$)	$(2.3 \pm 0.3 \pm 0.3) \times 10^{-5}$	[197]
$\frac{n_D}{n_H}$	1998	Ly-α observations Q1937-1009 (revised estimate)	$(3.24 \pm 0.3) \times 10^{-5}$	[185]
$\frac{n_D}{n_H}$	1998	Ly-α observations Q1009-2956 ($z = 2.504$)	$(3.39 \pm 0.25) \times 10^{-5}$	[184]
$\frac{n_D}{n_H}$	1997	Ly-α observations Q1718-4807 ($z = 0.701$)	$(20 \pm 5) \times 10^{-5}$	[183]
$\frac{n_{^3He}}{n_H}$	1992	Planetary nebulae	$\gtrsim 10^{-3}$	[191]
$\frac{n_{^3He}}{n_H}$	1997	Planetary nebulae	$[0.1, 1] \times 10^{-3}$	[190]
$\frac{n_{^3He}}{n_H}$	1994	Interstellar medium	$[1, 4] \times 10^{-4}$	[189]
$Y_{^4He}$	1995	49 low-metallicity, extragalactic H II regions	0.234 ± 0.003	[182]
$Y_{^4He}$	1998	45 low-metallicity, extragalactic H II regions	0.244 ± 0.002	[181]
$Y_{^4He}$	1994	10 low-metallicity, blue compact galaxies	0.240 ± 0.005	[180]
$\frac{n_{^7Li}}{n_H}$	1996	41 population II halo stars	$(1.729 \pm 0.011) \times 10^{-10}$	[196]

Computer programs for nucleosynthesis calculations

Wagoner et al. (1967, 1973)

- first numerical BBN code
- based on considerations of Peebles
- no longer maintained

Kawano (1988,1992)

- revised version of the Wagoner code
- $\sim$ 4800 lines F77 code
- thoroughly documented
- many modified versions

Fiorentini et al., Lisi et al. (1998,1999)

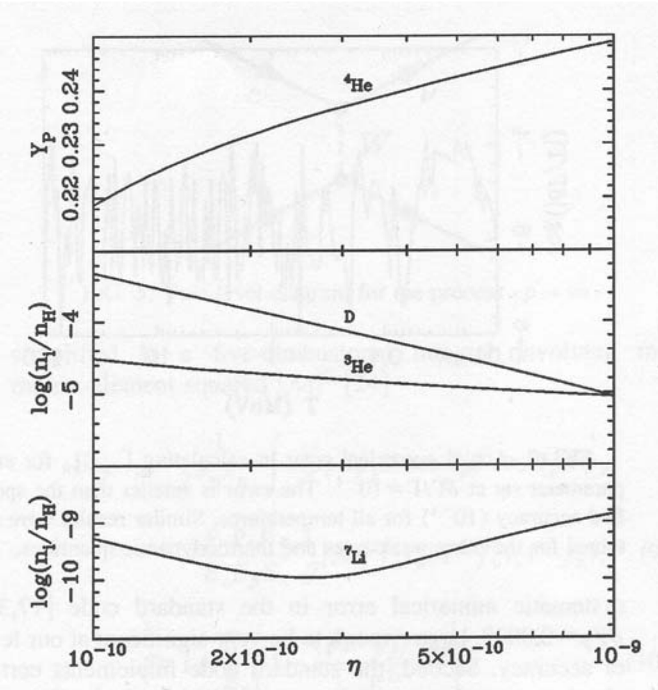
- $\sim$ 750 lines Fortran code
- well documented
- integrated χ^2 fitting
- based on results from Kawano code (+ corrections from Sakar)
- polynomial approximation

Lopez & Turner (1999)

- build from scratch
- take into account higher order effects (zero and finite temperature, Coulomb, finite nucleon-mass corr. to weak rates; QED corrections to plasma, m_e, T_ν; incomplete neutrino decoupling)
- $\Delta Y_{^4\mathrm{He}} < \pm 0.0002$
- not publically available (?)

Combined observations & Other groups

Ref.	Year	$\frac{n_{\mathbf{D}}}{n_{\mathbf{H}}}$	$\frac{n_{^3\mathbf{He}}}{n_{\mathbf{H}}}$	$Y_{^4\mathbf{He}}$	$\frac{n_{^7\mathbf{Li}}}{n_{\mathbf{H}}}$
[188]	1994	$(2.52 \pm 0.49) \times 10^{-5}$	$(1.48 \pm 0.13) \times 10^{-5}$	0.274 ± 0.016	
[187]	1995	$[1.9, 3.3] \times 10^{-5}$	$[1.9, 18] \times 10^{-5}$		
[186]	1996		$\lesssim 3.2 \times 10^{-5}$		
[161]	1999	$[2.9, 4] \times 10^{-5}$, $[1, 3] \times 10^{-4}$	$\sim 10^{-3}$	$[0.228, 0.248]$	$[1.2, 1.9] \times 10^{-10}$



Lopez, Turner (1999)

Theory

$Y_{^4\mathrm{He}} = [0.26, 0.28]$	Peebles (1966)
$Y_{^4\mathrm{He}} = [0.2, 0.3]$	Wagoner, Fowler, Hoyle (1967)
$Y_{^4\mathrm{He}} = [0.21, 0.24]$	Smith & Kawano (1993)
$Y_{^4\mathrm{He}} = 0.2462 \pm 0.0002$	Lopez & Turner (1999)

References

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