Foreword

This material was prepared by D. Puetzfeld as part of Iowa States Astro 405/505 (Fall 2004) course. Comments and suggestions are welcome! Please report any errors, typos, etc. (probably many) back to me (dpuetz@iastate.edu). You can find the material covered in the single lectures online under www.thp.uni-koeln.de/∼dp (click on teaching). There will also be some small computer algebra programs available on this site. The script only supplements the lecture, it is not a substitute for attending the lecture!

Warning: This is a "living" script, i.e. I will update it occasionally without notice.

Last update: 9th December 2004

that the metric is time-independent $g_{\mu\nu,0} = 0$[38]. Therefore the functions $v = v(r)$ and $\mu = \mu(r)$ will depend on the radial coordinates only. The next step is to calculate the Christoffel symbols for the metric[39], nowadays this can be easily done by the use of a computer algebra program, we only quote the final result[40]

$$\begin{aligned}
\Gamma^1_{00} &= \frac{1}{2}e^{\nu-\mu}\nu^{'}, \quad \Gamma^1_{11} = \frac{1}{2}\mu^{'}, \quad \Gamma^2_{33} = -\sin\theta\cos\theta, \\
\Gamma^1_{22} &= -re^{-\mu}, \quad \Gamma^2_{12} = \frac{1}{r}, \quad \Gamma^3_{23} = \frac{\cos\theta}{\sin\theta}, \\
\Gamma^0_{10} &= \frac{1}{2}\nu^{'}, \quad \Gamma^1_{33} = -re^{-\mu}\sin^2\theta, \quad \Gamma^3_{13} = \frac{1}{r}.
\end{aligned} \tag{1.27}$$

From the connection we can calculate the non-vanishing components of the Ricci tensor $R_{\mu\nu}$

$$R_{00} = e^{\nu-\mu}\left\{-\frac{\nu^{''}}{2} - \frac{\nu^{'}}{r} + \frac{\nu^{'}}{4}\left(\mu^{'} - \nu^{'}\right)\right\}, \tag{1.28}$$

$$R_{11} = \frac{\nu^{''}}{2} - \frac{\mu^{'}}{r} + \frac{\nu^{'}}{4}\left(\nu^{'} - \mu^{'}\right), \tag{1.29}$$

$$R_{22} = \frac{1}{\sin^2\theta}R_{33} = e^{-\mu}\left\{1 - e^{\mu} + \frac{r}{2}\left(\nu^{'} - \mu^{'}\right)\right\}. \tag{1.30}$$

Therefore the field equations for our spherically symmetric and stationary ansatz for the metric, remember $R_{\mu\nu} = 0$, turn out to be three equations for the two unknown functions (however there is no overdetermination because the equations are not independent, remember the identity $G^{\alpha\beta}{}_{;\beta} = 0 \Rightarrow \left(R^1{}_1 - R^0{}_0 - 2R^2{}_2\right)^{'} + \left(R^1{}_1 - R^0{}_0\right)\nu^{'} + \frac{4}{r}\left(R^1{}_1 - R^2{}_2\right)$). Subtracting (1.29) from (1.28) we find

$$\mu^{'} + \nu^{'} = 0 \Rightarrow \mu + \nu = C_1 = \text{const.} \tag{1.31}$$

Reinserting this into (1.30) we find $1 - e^{\mu} - r\mu^{'} = 0 \Leftrightarrow \left(re^{-\mu}\right)^{'} = 1 \Rightarrow re^{-\mu} = r + C_2$ (with $C_2 = \text{const}$). Renaming the second integration constant $C_2 := -2m$ we have $e^{-\mu} = 1 - \frac{2m}{r}$. Hence we can infer from (1.31) $e^{\nu} = e^{C_1-\mu} = e^{C_1}\left(1 - \frac{2m}{r}\right)$. Therefore the line element becomes $ds^2 = e^{C_1}\left(1 - \frac{2m}{r}\right)dt^2 - \left(1 - \frac{2m}{r}\right)^{-1}dr^2 - d\Omega^2$. Rescaling of the

[38] In this case the spacetime described by the metric has a fourth Killing vector. In general a spactime is called *stationary* if there exists one timelike ($X^{\mu}X_{\mu} > 0$) Killing vectorfield. In fact this vectorfield is also orthogonal to the hypersurface $t = \text{const}$ and therefore the corresponding spacetime is called *static*. Note that the assumption of stationarity is not necessary at this point. We could also go ahead and work out the field equations for two time dependent functions $\nu = \nu(t, r)$ and $\mu = \mu(t, r)$. The result that the corresponding spacetime is static is generally known under the name *Birkhoff theorem*, i.e. every spherically symmetric solution of the vacuum field equations is necessarily static.

[39] Exercise: Check this calculation by hand! After that use a computer algebra system to verify your result.

[40] $(\dots)^{'} := \frac{\partial}{\partial r}$

time coordinate[41] finally yields the *Schwarzschild solution*

$$ds^2 = \left(1 - \frac{2m}{r}\right) dt^2 - \left(1 - \frac{2m}{r}\right)^{-1} dr^2 - r^2 \left(d\theta^2 + \sin^2\theta d\phi^2\right). \quad (1.32)$$

This solution describes the exterior gravitational field outside of a spherically symmetric body. The solution has some interesting properties: (i) It depends only on one integration constant m, the meaning of this constant still has to be determined. (ii) The solution does not depend on the detailed distribution of matter inside the body. (iii) In the coordinates used the solution (1.32) becomes singular at the radius $r = 2m$.

1.3.3 Physical consequences

Let us now sketch the derivation of the trajectories of test particles in the spacetime described by the Schwarzschild line element in equation (1.32). The geodesic equation

$$\frac{d^2x^\lambda}{ds^2} + \Gamma^\lambda_{\mu\nu}\frac{dx^\mu}{ds}\frac{dx^\nu}{ds} = 0, \quad (1.33)$$

takes the form[42]

$$\begin{aligned}
\ddot{r} - \frac{1}{2}\nu'\dot{r}^2 - re^\nu\dot{\theta}^2 - re^\nu\sin^2\theta\dot{\phi}^2 + \frac{1}{2}e^{2\nu}\nu'\dot{t}^2 &= 0,\\
\ddot{\theta} + \frac{2}{r}\dot{r}\dot{\theta} - \sin\theta\cos\theta\dot{\phi}^2 &= 0,\\
\ddot{\phi} + \frac{2}{r}\dot{r}\dot{\phi} + \frac{2\cos\theta}{\sin\theta}\dot{\theta}\dot{\phi} &= 0,\\
\ddot{t} + \nu'\dot{r}\dot{t} &= 0.
\end{aligned} \quad (1.34)$$

Due to the spherical symmetry of the problem it is possible to consider only geodesics which lie in one plane, an advantageous choice is $\theta = \frac{\pi}{2}$, i.e. we confine ourselves to the equatorial plane. This simplifies the set in (1.34) considerably, i.e.%

$$\begin{aligned}
\ddot{r} - \frac{1}{2}\nu'\dot{r}^2 - re^\nu\dot{\phi}^2 + \frac{1}{2}e^{2\nu}\nu'\dot{t}^2 &= 0,\\
\ddot{\phi} + \frac{2}{r}\dot{r}\dot{\phi} &= 0,\\
\ddot{t} + \nu'\dot{r}\dot{t} &= 0.
\end{aligned} \quad (1.35)$$

Additionally, the four components of the geodesic equation are not independent. One can show that multiplication of (1.33) with $g_{\nu\mu}\dot{x}^\mu$ yields the first integral[43]

$$\begin{aligned}
\left(g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu\right)^{\cdot} &= 0 \Rightarrow g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu = \text{const} \Rightarrow^{\text{normalized}} g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu = 1\\
&\Leftrightarrow {}^{\text{SS inserted}} e^\nu\dot{t}^2 - e^{-\nu}\dot{r}^2 - r^2\dot{\phi}^2 = 1.
\end{aligned} \quad (1.36)$$

[41] Exercise: Write down this transformation explicitly. Check if the solution really satisfies the field equations (this can be done very quickly by the use of CA).

[42] $(\dots)^{\cdot} := \frac{d}{ds}$

[43] This integral has of course a physical interpretation if we think of a particle with velocity $u^\mu = \dot{x}^\mu$ moving on this geodesic.

Where in the last line we inserted the explicit form of the Schwarzschild line element (in order to avoid many terms we use $e^\nu = \left(1 - \frac{2m}{r}\right))$. From the last two equations in (1.35) we get the following integrals

$$r^2\dot{\phi} = a = \text{const}, \quad e^\nu \dot{t} = b = \text{const}, \tag{1.37}$$

the first of them representing the conservation of angular momentum of the particle and the second one corresponding the conservation of energy of the test particle in a time-independent field[44]. By combining the last two equations from (1.37) with the one in (1.36) and substituting back for e^ν we obtain

$$\left\{\frac{d}{d\phi}\left(\frac{1}{r}\right)\right\}^2 + \frac{1}{r^2} = \frac{b^2-1}{a^2} + \frac{2m}{a^2 r} + \frac{2m}{r^3}.$$

Comparison with Newtonian theory In order to compare this result with the Newtonian theory we differentiate this equation with respect to ϕ yielding

$$\frac{d^2}{d\phi^2}\left(\frac{1}{r}\right) + \frac{1}{r} = \frac{m}{a^2} + \frac{3m}{r^2}. \tag{1.38}$$

The corresponding Newtonian result for a central body with mass M is

$$\frac{d^2}{d\phi^2}\left(\frac{1}{r}\right) + \frac{1}{r} = \frac{GM}{a^2 c^2} \tag{1.39}$$

Comparison of the results in the last two equations determines the so-far unspecified integration constant within the Schwarzschild metric which is now given by $m = \frac{GM}{c^2}$. Hence m represents the mass of the central body in some relativistic units[45]. Additionally, we recognize that there will be an relativistic correction to the Newtonian result in form of the second term on the rhs of (1.38).

Some tests There are several test connected with the Schwarzschild metric: (i) The precession of the perihelia of the orbits of the inner planets $\delta\phi = \frac{6\pi m}{r_0(1-\epsilon^2)}$ radians/revolution. Here ϵ denotes the eccentricity and r_0 the semimajor axis of the orbit. Note that the positive sign in the formula denotes a precession in the same direction of the motion of the testparticle. For Mercury one has $\delta\phi = 0.1038''$ per revolution[46]. (ii) Deflection of light $\delta = \frac{2m}{r_0}$. For a light ray gracing the outer limb the sun we have

[44] At this point one should mention that the existence of any Killing vector of a space corrensponds to a first intergral of the geodesic equation, which is of course equivalent to a conservation law for the geodesic motion of a test particle.

[45] Exercise: Convince yourself that $[m]$= length. Verify the numerical values $m_{\text{sun}} \approx 1.5$ km, $m_{\text{earth}} \approx 0.5$ cm.

[46] Exercise: Derive the value of $\delta\phi$ for another planet in the solar of your choice. Why is it a good idea to pick mercury.

$r_0 \approx 7 \times 10^5$ km and $m \approx 1.5$ km, hence $\delta \approx 1.75''$.[47] Note that Newtonian gravity predicts only half of this value. (iii) The time delay in the field of massive body. This test was originally performed by studying reflected radar echoes from Mercury and Venus, approximately one has $\Delta t \approx 2 \times 10^{-4}$s.

Lightcones Light will move along lightlike curves, i.e. curves whose tangent vector satisfies $u^\mu u_u = 0$. From the line element in (1.32) we can infer that (in the plane $\dot{\theta} = \dot{\phi} = 0$)

$$\left(1 - \frac{2m}{r}\right) \dot{t}^2 - \left(1 - \frac{2m}{r}\right)^{-1} \dot{r}^2 = 0. \tag{1.40}$$

For the different regions we have[48]

$$\frac{dr}{dt} = \pm \left(1 - \frac{2m}{r}\right) = \pm \left\{ \begin{array}{ll} < 0 & \text{for } r > 2m \\ > 0 & \text{for } r < 2m \\ 0 & \text{for } r = 2m \end{array} \right| .$$

Of course this equation describes the opening angle of the light cones for outgoing (+) or ingoing (−) light signals. Observer that asymptotically we have the same behaviour as in Minkowski spacetime. Additionally there is a change in the character of the coordinates for values of r smaller/larger than the gravitational radius. If we compare the two different forms of the lightcones on either side of the surface $r = 2m$ we recognize immediately that the coordinate set does not cover the whole plane and therefore should be replaced by another system. A better coordinate system was suggested by Eddington and Finkelstein who devised the following transformations for the Schwarzschild time coordinate

$$t = \tilde{t} \pm 2m \ln \left| \frac{r}{2m} - 1 \right|, \; r = \tilde{r}, \; \theta = \tilde{\theta}, \; \phi = \tilde{\phi}.$$

With these relations the line element from (1.32) becomes non-diagonal, and we obtain two (depending on the sign) metrics

$$ds^2 = \left(1 - \frac{2m}{r}\right) d\tilde{t}^2 - \left(1 + \frac{2m}{r}\right) dr^2 \pm \frac{4m}{r} d\tilde{t} dr - r^2 \left(d\theta + \sin^2\theta d\phi^2\right). \tag{1.41}$$

Note that the metric coefficients remain regular at $r = 2m$ and the metric has the same asymptotic behaviour as the Schwarzschild metric. Now lets investigate again the structure of the null cones of this metric. For fixed θ and ϕ we have

$$\left(1 - \frac{2m}{r}\right) d\tilde{t}^2 - \left(1 + \frac{2m}{r}\right) dr^2 \pm \frac{4m}{r} d\tilde{t} dr = 0.$$

[47] Exercise: Calculate the value of the the maximal deflection for some other astrophysical objects (Examples: Earth, a neutron star, a supermassive black hole).

[48] Exercise: Show that one can derive $t = \pm\left(r + 2m \ln |r - 2m| + \text{const}\right)$ from equation (1.40). Plot these lines in the (r, t) plane for different values of the integration constant. What does this tell you about the coordinate time?

Selecting the $-$ sign this leads to two for the radial null directions

$$\frac{dr}{d\tilde{t}} = -1, \; \frac{dr}{d\tilde{t}} = \frac{r-2m}{r+2m}. \tag{1.42}$$

The first solution represents straight lines which cover the complete coordinate and describes radially ingoing light signals and behave completely regular at $r = 2m$. The second solution, which describes radially outgoing signals, has three distinct values, i.e. $\frac{dr}{d\tilde{t}} = \begin{cases} -1 & \text{for } r \to 0 \\ 0 & \text{for } r \to 2m \\ 1 & \text{for } r \to \infty \end{cases}$. Hence for outgoing signal the coordinate system has the same problems as the original Schwarzschild version of the metric[49]. But if we consider the orientation of the light cones described by (1.42) it becomes clear that the hypersurface at $r = 2m$ represents some kind of semipermeable membrane for photons (and therefore also for observers who move on timelike curves). The hypersurface at $r = 2m$ is called an *event horizon*, since it is not possible to send some light signal to the region $r > 2m$ as soon as one has crossed the surface at $r = 2m$[50]. The + sign in (1.41) describes the time reversed case. Without going into further detail we note there exist better[51], i.e. geodesically complete, coordinates for the Schwarzschild geometry which avoid the problems encountered at $r = 2m$. Finally, we remark that there is another way to verify that the singularity at $r = 2m$ is only a coordinate singularity, namely by calculating the curvature invariant $R^{\alpha\beta\mu\nu}R_{\alpha\beta\mu\nu} = \frac{48m^2}{r^6}$ which shows that the spacetime described by (1.32) exhibits only an essential singularity at $r = 0$.

Interior and other exact solutions Keep in mind that we have only discussed the exterior, i.e. vacuum solution, of the field equations (1.25) in the spherically symmetric case. Of course people also modeled the interior of a star in general relativistic setup. This is in general a very complicated problem since one has to incorporate the processes which take place in a realistic star. Nevertheless people have studied highly symmetric models and found exact solutions of the field equations of GR, one of these was given by Schwarzschild in 1916 in which he assumes that the interior of the star is made of an incompressible fluid. This is the so-called *interior* Schwarzschild solution, we will not discuss this solution here.

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[49] Exercise: Show that for outgoing null-curves and $-$ sign in the metric one has $t = r + 4m \ln|2m - r|$ +const. Work out and plot the parametric lines $t(r)$ for both cases ($\pm$) of the Eddington-Finkelstein metric.

[50] The surface M at $r = 2m$ parametrized by

$$f(x^\mu) = r - 2m = 0,$$

has the normal vector $n_\mu = f_{,\mu} = (0,1,0,0)$ and is therefore a *null-hypersurface* or *characteristic-surface*, since $n_\mu n^\mu = 0$ on M. Exercise: Work out the contravariant components $g^{\alpha\beta}$ of the metric the Eddington-Finkelstein metric and verify the statement from above.

[51] You can find the discussion in every textbook on GR $\to$ Kruskal-Szekeres coordinates.

1.4 Part IV - Basic assumptions in cosmology

Goal: Sketch fundamental ideas which form the basis of modern cosmology.

1.4.1 Two assumptions

Without going into observational detail we mention here only two assumptions which lead us to consider models of the FLRW type: (i) a global expansion and (ii) the homogeneity and isotropy of space. The first assumption goes back to an observation of Hubble, who found, by means of measuring the redshift and the luminosity of extragalactic nebulae, a linear relationship between the radial velocity v and the distance r ascribed to the nebulae with respect to an earthbound observer. This relationship is expressed in the famous Hubble law

$$v = H_0\, d, \tag{1.43}$$

where H_0 denotes the so-called Hubble constant. Let us stress that this does *not* single out a preferred observer in the universe as one might intuitively guess. The velocity v in the above formula arises from the original interpretation of the observed redshift as Dopplershift. In the context of General Relativity (GR) this observation can also be interpreted as a global expansion of the spacetime. The notion *redshift*, associated with the global expansion, and the notion *Dopplershift*, associated with the peculiar motion of stars, are often used synonymously in astrophysical context. However they are completely different physical effects. In order to take care of the first assumption (i), a cosmological model should incorporate something like a global scale factor $S(t)$, which describes the size of the universe. The second assumption (ii) relies on the fact that matter seems to be distributed very homogeneously in the universe at least in a statistical manner. These two assumptions motivate our ansatz for the metric and the energy-momentum in the next section.

The Robertson-Walker metric The assumption of homogeneity and isotropy and global expansion leads to the so-called Robertson-Walker metric. Using spherical coordinates (t, r, θ, ϕ) the line element is given by

$$ds^2 = dt^2 - S^2 \left(\frac{1}{1-kr^2}\, dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 \right). \tag{1.44}$$

The function $S(t)$ represents the cosmic scale factor, and k can be chosen[52] to be $+1$, -1, or 0 for spaces with constant positive, negative, or zero spatial curvature, respectively.[53]

[52] After an appropriate rescaling of the coordinates.

[53] Exercise: Show that the Roberston-Walker metric is spherically symmetric. Provide the explicit form of the three Killing VFs corresponding to the spherical symmetry.

The Energy momentum tensor of an ideal fluid As already mentioned above on large scales matter appears to be distributed in a homogeneous and isotropic way. Therefore we shall assume that the (average) matter tensor $T_{\mu\nu}$ has the simple form

$$T_{\mu\nu} = (\rho + p)\, u_\mu u_\nu - p g_{\mu\nu}, \tag{1.45}$$

i.e. describes a perfect fluid. The two quantities $\rho = \rho(t)$ and $p = p(t)$ correspond to the energy density and pressure measured in some frame. A particularly well adapted frame is the comoving frame. In this frame the four velocity in (1.45) is reduced to $u^\mu = (1, 0, 0, 0)$[54]. We will take this frame when we work out the field equations in the next section.

1.4.2 The Friedmann equations

From the ansatz for the metric (1.44) and the energy-momentum (1.45) in we can immediately work out the form of the field equations

$$\left(\frac{\dot{S}}{S}\right)^2 + \frac{k}{S^2} - \frac{\Lambda}{3} = \frac{\kappa}{3}\rho, \tag{1.46}$$

$$2\frac{\ddot{S}}{S} + \left(\frac{\dot{S}}{S}\right)^2 + \frac{k}{S^2} - \Lambda = -\kappa p, \tag{1.47}$$

and recover the well known form of the so-called *Friedmann* equations. Hence the field equations (1.25), note that we did not assume that the cosmological constant vanishes, turn into a set of ordinary differential equations for the scale factor $S(t)$. The functions ρ, p and the parameters k, Λ depend on the model we decide to consider. Note that ρ and p are related by an equation of state $p = p(\rho)$, $p = \frac{1}{3}\rho$ in case of a radiation-dominated universe, e.g.

Epochs During its evolution the universe goes through different epochs, that are characterized by the respective equation of state. The inspection of (1.46)–(1.47) reveals that the solution for the scale factor $S(t)$ depends on the choice of this equation of state. In addition to the field equations we have one differential identity (remember the definition of the Einstein tensor), i.e.%

$$T^{\mu\nu}{}_{;\nu} = 0. \tag{1.48}$$

Let us assume that the equation of state takes the form $p(t) = w\,\rho(t)$, with $w =$ const. Using (1.44), and (1.45), equation (1.48) turns into

$$\dot{\rho} S = -3\dot{S}\,(\rho + p) \quad \overset{p=w\rho}{\Rightarrow} \quad \rho = \varkappa_w S^{-3(1+w)} \sim S^{-3(1+w)}, \tag{1.49}$$

[54] In this case the velocity shall be normalized according to $g_{\mu\nu}u^\mu u^\nu = 1$.

where $\varkappa_w$ is an integration constant. Thus, we have found a relation between the energy density and the scale factor, which depends on the constant w from the equation of state. Substituting back (1.49) into (1.46) yields

$$\left(\frac{\dot{S}}{S}\right)^2 + \frac{k}{S^2} - \frac{\lambda}{3} = \frac{\kappa}{3}\varkappa_w S^{-3(1+w)}. \tag{1.50}$$

Special case A frequently discussed case is the spatially flat one with vanishing cosmological constant. Ignoring all emerging constants in the solution for $S(t)$, equation (1.50) yields

$$\left(\frac{\dot{S}}{S}\right)^2 = \frac{\kappa}{3}\varkappa_w S^{-3(1+w)} \quad \overset{\Lambda=k=0,\, \varkappa_w=1,\, p=w\rho,\, w\neq-1}{\Longrightarrow} \quad S \sim t^{\frac{2}{3(1+w)}}. \tag{1.51}$$

Bibliography

[**Beware!**] There is an enormous amount of books on relativity and cosmology, the bibliography (which is in random order) is not intended to be complete in any sense. It should only be used as a guide, which book is suitable for you depends on your taste and learning style!

[**Relativity**]

[1] W. Rindler: *Introduction to Special relativity*, Oxford University Press, Oxford (1991)
Very clear and concise introduction to SR, suitable for self-study, good book for beginners.

[2] R.U. Sexl, H.K. Urbantke: *Relativity, Groups, Particles: Special Relativity and Relativistic Symmetry in Field and Particle Physics*, Springer, New York (2001)
Contains an introduction to SR, covers many advanced topics in SR, good for people who want to learn more about the group structure of SR, maybe not the best choice for a beginner who looks for a book for self-study.

[3] W. Rindler: *Essential relativity*, Springer, New-York (1977)
Covers SR and GR, has a basic introduction to cosmology, very clear and concise, suitable for self-study, good book for beginners.

[4] R. D'Inverno: *Introducing Einstein's Relativity*, Oxford University Press, Oxford (1992)
Very good self contained introduction to GR, many pictures, suitable for self-study, good book for beginners.

[5] R. M. Wald: *General Relativity*, University of Chicago Press, Chicago (2002)
Very good self contained introduction to GR, covers also some more advanced topics, suitable for self-study but maybe not the book every beginner wants to start with.

[6] S. Weinberg: *Gravitation and cosmology.* Wiley, New York (1972)
Introduction to many topics in GR and cosmology, covers also some more advanced topics, maybe not the book every beginner wants to start with.

[7] C.W. Misner, K.S. Thorne, J.A. Wheeler: *Gravitation.* Freeman, San Francisco (1973)
Encyclopedic book on many aspects of gravitation, covers also advanced topics, suitable for self-study of you have enough time, maybe not the book every beginner wants to start with.

[**Cosmology**]

[8] P.J.E. Peebles: *Principles of physical cosmology.* Princeton University Press, Princeton (1993)
Covers many topics of cosmology in a non-technical way, very good for beginners and for self-study.

[9] J.A. Peacock: *Cosmological physics.* Cambridge University Press, Cambridge (1999)
Covers many topics in cosmological physics, includes very short introductions to GR as well as QFT, more suitable for reference not for self-study.

[10] E.W. Kolb, M. S. Turner: *The early universe.* Addison-Wesley, Redwood City (1990)
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[11] T. Padmanabhan: *Theoretical astrophysics Volume I: Astrophysical processes.* Cambridge University Press, Cambridge (2000)
T. Padmanabhan: *Theoretical astrophysics Volume II: Stars and stellar systems.* Cambridge University Press, Cambridge (2001)
T. Padmanabhan: *Theoretical astrophysics Volume III: Galaxies and cosmology.* Cambridge University Press, Cambridge (2002)
Three volume course on astrophysics and cosmology, covers many topics, very good for reference, maybe not the best book for self-study.

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